

AA4: Solving Logarithm and Exponential Equations

Solve each equation for x. Check your answers and remove any extraneous solutions.

1. $2^{4x-2} = 64$

$$\begin{aligned} \log_2 \log_2 \\ 4x-2 &= 6 \\ +2 \quad +2 \\ 4x &= 8 \\ \frac{4}{4} \quad \frac{4}{4} \\ x &= 2 \end{aligned}$$

2. $125^x = 25$

$$\begin{aligned} \log_{125} \log_{125} \\ x &= \frac{2}{3} \end{aligned}$$

3. $9^{x+2} = \frac{1}{27}$

$$\begin{aligned} \log_9 \log_9 \\ x+2 &= -1.5 \\ -2 \quad -2 \\ x &= -3.5 \end{aligned}$$

4. $8^x = 12,143$

$$\begin{aligned} \log_8 \log_8 \\ x &= \frac{\log(12143)}{\log(8)} \\ x &= 4.52 \end{aligned}$$

5. $9(4^x) = 36,864$

$$\begin{aligned} \frac{9}{9} \quad \frac{9}{9} \\ 4^x &= 4096 \\ \log_4 \log_4 \\ x &= 6 \end{aligned}$$

6. $2^{x-1} + 3 = 131$

$$\begin{aligned} -3 \quad -3 \\ 2^{x-1} &= 128 \\ \log_2 \log_2 \\ x-1 &= 7 \\ x &= 8 \end{aligned}$$

7. $5^{4x+2} = 37,500$

$$\begin{aligned} \log_5 \log_5 \\ 4x+2 &= \frac{\log(37500)}{\log(5)} \\ 4x+2 &= 6.54 \\ \frac{4x}{4} \quad \frac{2}{4} \quad \frac{2.54}{4} \\ x &= 1.135 \end{aligned}$$

8. $5(3^{x+4}) = 320$

$$\begin{aligned} \frac{5}{5} \quad \frac{5}{5} \\ 3^{x+4} &= 64 \\ \log_3 \log_3 \\ x+4 &= 3.79 \\ -4 \quad -4 \\ x &= -.21 \end{aligned}$$

9. $16^{4x-2} = \frac{1}{64}$

$$\begin{aligned} \log_{16} \log_{16} \\ 4x-2 &= -1.5 \\ 4x &= .5 \\ x &= .125 \end{aligned}$$

10. $\log_4(3x-5) = 3$

$$\begin{aligned} \frac{4}{4} \quad \frac{4}{4} \\ 3x-5 &= 4^3 \\ 3x-5 &= 64 \\ +5 \quad +5 \\ 3x &= 69 \\ \frac{3x}{3} \quad \frac{69}{3} \\ x &= 23 \end{aligned}$$

11. $3 + 4\log_9(2x) = 15$

$$\begin{aligned} -3 \quad -3 \\ 4\log_9(2x) &= 12 \\ \frac{4}{4} \quad \frac{4}{4} \\ \log_9(2x) &= 3 \\ \frac{\log_9(2x)}{9} \quad \frac{3}{9} \\ 2x &= 729 \\ \frac{2x}{2} \quad \frac{729}{2} \\ x &= 364.5 \end{aligned}$$

12. $2\log_5(3x) = 250$

$$\begin{aligned} \frac{2}{2} \quad \frac{2}{2} \\ \log_5(3x) &= 125 \\ \frac{\log_5(3x)}{5} \quad \frac{125}{5} \\ 3x &= 2.35 \times 10^{87} \\ \frac{3x}{3} \quad \frac{2.35 \times 10^{87}}{3} \\ x &= 7.84 \times 10^{86} \end{aligned}$$

Beware of spicyness....

13. $\log_2(x^2 - 9) = 4$

$$\begin{aligned} \frac{2}{2} \quad \frac{2}{2} \\ x^2-9 &= 16 \\ x^2 &= 25 \\ x &= \pm 5 \end{aligned}$$

14. $\log_3((x-1)(x+2)) = 2$

$$\begin{aligned} \frac{3}{3} \quad \frac{3}{3} \\ (x-1)(x+2) &= 9 \\ x^2+2x-1x-2 &= 9 \\ x^2+x-11 &= 0 \\ (x+.5)^2 - 11.25 &= 0 \\ (x+.5)^2 &= 11.25 \end{aligned}$$

15. $\log_{10}(x(x+15)) = 2$

$$\begin{aligned} \frac{10}{10} \quad \frac{10}{10} \\ x(x+15) &= 100 \\ x^2+15x-100 &= 0 \\ (x+20)(x-5) &= 0 \\ x &= -20, x = 5 \end{aligned}$$

Find the inverse of each function. Check your answer by graphing, using a table, or function composition.

1. $f(x) = 3^x$

$$y = 3^x$$

$$\log_3 y = x$$

$$\log_3 x = f^{-1}(x)$$

2. $g(x) = \log_3(x)$

$$y = \log_3(x)$$

$$3^y = x$$

$$3^x = g^{-1}(x)$$

3. $h(x) = 5(3)^x$

$$y = 5(3)^x$$

$$\log_3 \frac{y}{5} = x$$

$$\log_3 \left(\frac{y}{5} \right) = x$$

$$\log_3 \left(\frac{x}{5} \right) = h^{-1}(x)$$

4. $a(x) = \frac{5 \log_6(x+7)}{5}$

$$\frac{y}{5} = \log_6(x+7)$$

$$6^{\frac{y}{5}} = x+7$$

$$6^{\frac{y}{5}} - 7 = x$$

$$6^{\frac{x}{5}} - 7 = a^{-1}(x)$$

5. $b(x) = \frac{7(9)^{x+5}}{7}$

$$\log_9 \frac{y}{7} = x+5$$

$$\log_9 \left(\frac{y}{7} \right) = x+5$$

$$\log_9 \left(\frac{y}{7} \right) - 5 = x$$

$$\log_9 \left(\frac{x}{7} \right) - 5 = b^{-1}(x)$$

6. $c(x) = \frac{12 \log_5(x) - 7}{+7}$

$$\frac{y+7}{12} = \log_5(x)$$

$$5^{\frac{y+7}{12}} = x$$

$$5^{\frac{y+7}{12}} = x$$

$$5^{\frac{x+7}{12}} = c^{-1}(x)$$

7. $d(x) = \frac{.85(1.35)^{2x+3} - 5}{+5}$

$$\frac{y+5}{.85} = \frac{.85(1.35)^{2x+3}}{.85}$$

$$\log_{1.35} \frac{y+5}{.85} = 2x+3$$

$$\log_{1.35} \left(\frac{y+5}{.85} \right) = 2x+3$$

$$\log_{1.35} \left(\frac{y+5}{.85} \right) - 3 = 2x$$

$$\frac{\log_{1.35} \left(\frac{y+5}{.85} \right) - 3}{2} = d^{-1}(x)$$

8. $e(x) = 0.125 \log_{1.25}(3.5x) - 5$

$$e^{-1}(x) = \frac{1.25^{\frac{x+5}{0.125}}}{3.5}$$

9. $p(x) = 1234(5)^{6x+7} - 8$

$$p^{-1}(x) = \frac{\log_5 \left(\frac{x+8}{1234} \right) - 7}{6}$$