

Solving Radical Equations

Radical equations are equations that involve roots (square roots, cube roots, 4th roots, etc.)

Squaring undoes square roots. Cubing undoes cube roots. Raising both sides to an exponent undoes the corresponding root.

Because we are solving by undoing, we use SADMEP order.

Solve these problems side by side and notice how similar the processes are:

$3(x+5)^2 - 5 = 7$ $\begin{array}{r} +5 \\ +5 \end{array}$ $\frac{3(x+5)^2}{3} = \frac{12}{3}$ $(x+5)^2 = 4$ $\begin{array}{l} \swarrow \quad \searrow \\ x+5=2 \quad x+5=-2 \\ \begin{array}{r} -5 \\ -5 \end{array} \quad \begin{array}{r} -5 \\ -5 \end{array} \end{array}$	$3\sqrt{x+5} - 5 = 7$ $\begin{array}{r} +5 \\ +5 \end{array}$ $\frac{3\sqrt{x+5}}{3} = \frac{12}{3}$ $(\sqrt{x+5})^2 = 4^2$ $\begin{array}{r} x+5=16 \\ -5 \quad -5 \end{array}$ $x=11$
<p>Check your solution(s)</p> $3(-3+5)^2 - 5 \stackrel{?}{=} 7$ $3(2)^2 - 5 = 7$ $3(4) - 5 = 7$ $12 - 5 = 7$	<p>Check your solution(s)</p> $3\sqrt{11+5} - 5 = 7$ $3\sqrt{16} - 5 = 7$ $3(4) - 5 = 7$ $12 - 5 = 7$

Solve these problems side by side and notice how similar the processes are:

$-7(x-5)^3 + 8 = -48$ $\begin{array}{r} -8 \\ -8 \end{array}$ $\frac{-7(x-5)^3}{-7} = \frac{-56}{-7}$ $\sqrt[3]{(x-5)^3} = \sqrt[3]{8}$ $\begin{array}{r} x-5=2 \\ +5 \quad +5 \end{array}$ $x=7$	$-7\sqrt[3]{x-5} + 8 = -48$ $\begin{array}{r} -8 \\ -8 \end{array}$ $\frac{-7\sqrt[3]{x-5}}{-7} = \frac{-56}{-7}$ $\sqrt[3]{x-5} = 8$ $\begin{array}{r} x-5=512 \\ +5 \quad +5 \end{array}$ $x=517$
<p>Check your solution(s)</p> $-7(7-5)^3 + 8 = -48$ $-7(2)^3 + 8$ $-7(8) + 8$ $-56 + 8 = -48$	<p>Check your solution(s)</p> $-7\sqrt[3]{517-5} + 8 = -48$ $-7\sqrt[3]{512} + 8$ $-7 \cdot 8 + 8$ $-56 + 8 = -48$

Remember to follow SADMEP order (1. Undo **S**ubtraction/Addition, 2. Undo **D**ivision/Multiplication, 3. Undo **E**xponents/Radicals, 4. Undo the **P**arentheses/Grouping).

SADMEP helps you take the equation apart one step at a time. Plug your solution(s) back in and follow PEMDAS order to check your answer.

Solve the following equations and check your solution(s).

$\sqrt{x} + 5 = 10$ $\begin{array}{r} -5 \quad -5 \\ \sqrt{x} = 5 \end{array}$ $\sqrt{x^2} = 5^2$ $x = 25$	$\sqrt{x-3} - 3 = 3$ $\begin{array}{r} +3 \quad +3 \\ \sqrt{x-3} = 6 \end{array}$ $x-3 = 36$ $\begin{array}{r} +3 \quad +3 \\ x = 39 \end{array}$	$2\sqrt[3]{x+1} = 6$ $\begin{array}{r} \frac{2}{2} \quad \frac{2}{2} \\ \sqrt[3]{x+1} = 3 \end{array}$ $3 \sqrt[3]{x+1} = 3^3$ $x+1 = 27$ $\begin{array}{r} -1 \quad -1 \\ x = 26 \end{array}$
$\sqrt{2x+5} = 4$ $2x+5 = 16$ $\begin{array}{r} -5 \quad -5 \\ 2x = 11 \end{array}$ $\frac{2x}{2} = \frac{11}{2}$ $x = 5.5$	$3\sqrt{x-5} + 7 = 16$ $\begin{array}{r} -7 \quad -7 \\ 3\sqrt{x-5} = 9 \end{array}$ $\frac{3\sqrt{x-5}}{3} = \frac{9}{3}$ $\sqrt{x-5} = 3$ $x-5 = 9$ $\begin{array}{r} +5 \quad +5 \\ x = 14 \end{array}$	$4\sqrt{x-4} + 4 = 12$ $\begin{array}{r} -4 \quad -4 \\ 4\sqrt{x-4} = 8 \end{array}$ $\frac{4\sqrt{x-4}}{4} = \frac{8}{4}$ $\sqrt{x-4} = 2$ $x-4 = 16$ $\begin{array}{r} +4 \quad +4 \\ x = 20 \end{array}$
$-5\sqrt{x-4} + 7 = -3$ $\begin{array}{r} -7 \quad -7 \\ -5\sqrt{x-4} = -10 \end{array}$ $\frac{-5\sqrt{x-4}}{-5} = \frac{-10}{-5}$ $\sqrt{x-4} = 2$ $x-4 = 4$ $\begin{array}{r} +4 \quad +4 \\ x = 8 \end{array}$	<p>Challenge:</p> $\sqrt{x+2} - x = 2$ $\begin{array}{r} +x \quad +x \\ \sqrt{x+2} = (x+2)^2 \end{array}$ $x+2 = (x+2)(x+2)$ $x+2 = x^2 + 4x + 4$ $0 = x^2 + 3x + 2$ $0 = (x+2)(x+1)$ $x = -2 \quad x = -1$	<p>Challenge:</p> $\sqrt{x+1} - 2x = 3 - x$ $\begin{array}{r} +2x \quad +2x \\ \sqrt{x+1} = (3+x)^2 \end{array}$ $x+1 = x^2 + 6x + 9$ $0 = x^2 + 5x + 8$ $x = \frac{-5 \pm \sqrt{5^2 - 4(1)(8)}}{2(1)}$ <p>No solution.</p>

Write yourself a note about what you have learned about solving radical equations:

Use SADMEP order and undo the radical with the corresponding exponents.