

A. Linear Inequalities

- a. Consider the inequality $10x - 2 \leq 18$. Find 3 solutions to this inequality. In other words, find 3 values of x that make the inequality true.

$$10x - 2 \leq 18$$

$$\begin{array}{r} +2 \\ +2 \end{array}$$

$$10x \leq 20 \rightarrow x \leq 2$$

$x = 0, x = -10, x = -11111$

- b. What is the largest value of x that solves the inequality? How do you know it is the largest possible solution? Is there a smallest possible solution? Why or why not?



- c. Now consider the inequality $12 - 3x \leq 15$. Find 3 solutions to this inequality.

$$12 - 3x \leq 15$$

$$\begin{array}{r} -12 \\ -12 \end{array}$$

$$-3x \leq 3$$

$$\begin{array}{r} \div 3 \\ \div 3 \end{array}$$

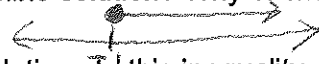
$$-x \leq 1$$

$$\begin{array}{r} \times -1 \\ \times -1 \end{array}$$

$$x \geq -1$$

$x = 0, x = 10, x = 100000$

- d. What is the smallest value of x that solves the inequality above. How do you know it is the smallest possible solution? Is there a largest possible solution? Why or why not?



- e. Now consider the inequality $-3 \leq 4 - x \leq 2$. Find 3 solutions to this inequality.

$$-3 \leq 4 - x \leq 2$$

$$\begin{array}{r} -4 \\ -4 \end{array}$$

$$-7 \leq -x \leq -2$$

$$\begin{array}{r} \div -1 \\ \div -1 \end{array}$$

$$7 \geq x \geq 2$$

$x = 2, x = 3, x = 4$

- f. What is the smallest value of x that solves the inequality above. How do you know it is the smallest possible solution? Is there a largest possible solution? Why or why not?

$x = 7 = \text{largest}$, $x = 2 = \text{smallest}$



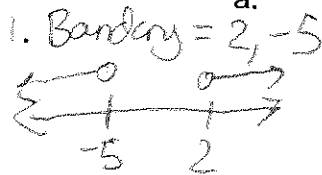
A **boundary point** is the smallest (or largest depending on whether it is a $<$ or $>$ problem) solution to an inequality. A **test point** can be used to check to see where solutions lie relative to the boundary point. How can you use the boundary and test points to write all solutions to an inequality? See Solving Linear Inequalities for further guidance.

- e. Practice: Solve each inequality below. Write your solution using inequality symbols ($<$, $>$, \leq , \geq) and graphically on a number line.

- $5 - 2x \geq 1 - 7x$
 $\begin{array}{r} +7x \\ +7x \end{array}$
 $5 + 5x \geq 1 - 5$
 $\begin{array}{r} -5 \\ -5 \end{array}$
 $5x \geq -4$
 $\begin{array}{r} \div 5 \\ \div 5 \end{array}$
 $x \geq -\frac{4}{5}$
- $4x - 3 < 10x + 21$
 $\begin{array}{r} -10x \\ -10x \end{array}$
 $-6x - 3 < 21$
 $\begin{array}{r} +3 \\ +3 \end{array}$
 $-6x < 24$
 $\begin{array}{r} \div -6 \\ \div -6 \end{array}$
 $x > -4$
- $2(x - 1) - 3 > -11$
 $\begin{array}{r} +3 \\ +3 \end{array}$
 $2(x - 1) > -8$
 $\begin{array}{r} \div 2 \\ \div 2 \end{array}$
 $x - 1 > -4$
 $\begin{array}{r} +1 \\ +1 \end{array}$
 $x > -3$
- $5 - (x - 3) \leq 18$
 $\begin{array}{r} -5 \\ -5 \end{array}$
 $-(x - 3) \leq 13$
 $\begin{array}{r} -1 \\ -1 \end{array}$
 $x - 3 \geq -13$
 $\begin{array}{r} +3 \\ +3 \end{array}$
 $x \geq -10$
- $4 \leq 10 - 2x \leq 18$
 $\begin{array}{r} -10 \\ -10 \end{array}$
 $\begin{array}{r} -10 \\ -10 \end{array}$
 $-6 \leq -2x \leq 8$
 $\begin{array}{r} \div -2 \\ \div -2 \end{array}$
 $3 \geq x \geq -4$

B. Quadratic Inequalities

- a. Consider the inequalities below:



i. $(x - 2)(x + 5) > 0$

ii. $2(x - 4)^2 + 3 \leq 53$

iii. $x^2 + 4x < -3$

How could you find the boundary points for each inequality? How many boundary points will these inequalities have? How many test points do you need to use in this case?

Solve by factoring/CTS & SADMED, 2 Boundary, 3 Test.

- b. Use the boundary and test points to write the solutions to these inequalities.

Solve each inequality by finding **boundary points** and using **test points** to determine the solution intervals. (see example below if you get stuck)

See next pages for solutions.

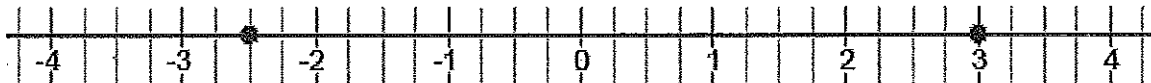
Example: Solve $(x - 3)(2x + 5) > 0$

Step 1: Boundary point: Solve $(x - 3)(2x + 5) = 0$

Using the Zero Product Property: $x - 3 = 0$ or $2x + 5 = 0$

$$x = 3 \quad \text{or} \quad x = -2.5$$

So the boundary points are at $x = 3$ or $x = -2.5$



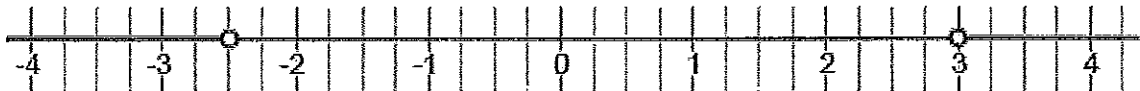
Step 2: Test points: Choose a value in each possible solution interval.

$-2.5 > x$: $x = -3$ $(-3 - 3)(2 \cdot -3 + 5) = (-6)(-6 + 5) = (-6)(-1) = 6 > 0$ True

$-2.5 < x < 3$: $x = 0$ $(0 - 3)(2 \cdot 0 + 5) = (-3)(0 + 5) = (-3)(5) = -15 > 0$ False

$x > 3$: $x = 4$ $(4 - 3)(2 \cdot 4 + 5) = (1)(8 + 5) = (1)(13) = 13 > 0$ True

So values in the first and third intervals make the inequality true. The solutions are $-2.5 > x$ or $x > 3$



Practice:

1. $(x + 11)(x - 2) \leq 0$

2. $(x - 4)(x + 6) \geq 0$

3. $(x - 4)^2 + 8 < 17$

4. $3(x + 5)^2 - 1 \leq 11$

5. $x^2 + 4x - 12 > 0$

6. $2x^2 - 5x < 12$

C. Other functions

a. Consider the inequalities below:

i. $2\sqrt{x - 3} + 1 > 5$

ii. $-4|x - 3| - 2 \leq -10$

iii. $\frac{4}{x} - 2 \leq \frac{5}{2x}$

How could you find the boundary points for each inequality? How many boundary points will each type of inequality have?

b. Use the boundary and test points to write the solutions to these inequalities. See Absolute Value Inequality for further guidance.

c. For problem a.i., why does it make sense that the solution should be in the form $3 \leq x < \#$? Explain your answer fully.

d. Extra practice

• $2(x - 1)^3 - 1 \geq 3$ $(x - 1)^3 \geq 2$

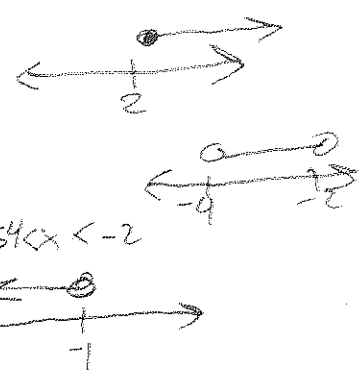
• $|x + 3| - 1 < 0$ $2(x - 1)^3 \geq 4$ $x - 1 \geq \sqrt[3]{2}$ $x \geq \sqrt[3]{2} + 1$

• $-2\sqrt[3]{x} + 7 > 9$ $|x + 3| < 1 \rightarrow |x + 3| < 1 \rightarrow -4 < x < -2$

• $\frac{2}{x+1} - 1 \leq \frac{5}{x-1}$ $-2\sqrt[3]{x} > 2 \rightarrow \sqrt[3]{x} < -1$ $x < -1$

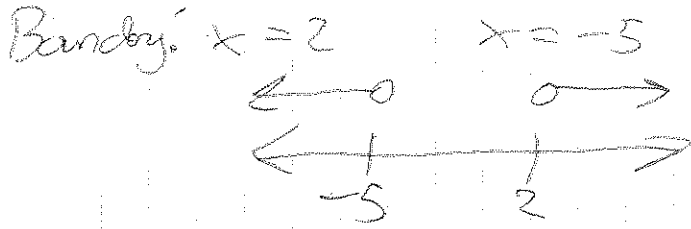
• $\frac{2}{x+1} - x \leq \frac{4}{x-1}$

$\frac{2}{x+1} - 1 \leq \frac{5}{x-1}$
 $\frac{2(x-1) - (x-1)(x+1)}{(x+1)(x-1)} \leq \frac{5}{x-1}$
 $\frac{2x-2 - (x^2-1)}{(x+1)(x-1)} \leq \frac{5}{x-1}$
 $\frac{2x-2-x^2+1}{(x+1)(x-1)} \leq \frac{5}{x-1}$
 $\frac{-x^2+2x-1}{(x+1)(x-1)} \leq \frac{5}{x-1}$
 $0 = x^2 + 3x + 6$



Make

B) i) $(x-2)(x+5) > 0$



Test: $x=-10$ | $x=0$ | $x=10$

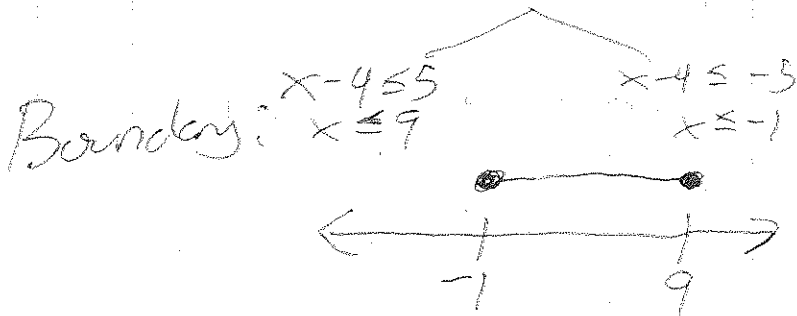
$(-10-2)(-10+5)$ $(-12)(-5)$ $60 > 0$ ✓	$(0-2)(0+5)$ $(-2)(5)$ $-10 > 0$	$(10-2)(10+5)$ $(8)(15)$ $120 > 0$ ✓
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ii) $2(x-4)^2 + 3 \leq 53$

$$\frac{2(x-4)^2}{2} \leq \frac{50}{2}$$

$$\sqrt{(x-4)^2} \leq \sqrt{25}$$

$$x-4 \leq \pm 5$$



Test: $x=-10$ | $x=0$ | $x=10$

$2(-10-4)^2 + 3$ $2(-14)^2 + 3$ $2(196) + 3$ $392 + 3$ $395 \leq 53$	$2(0-4)^2 + 3$ $2(-4)^2 + 3$ $2(16) + 3$ $32 + 3$ $35 \leq 53$ ✓	$2(10-4)^2 + 3$ $2(6)^2 + 3$ $2(36) + 3$ $72 + 3$ $75 \leq 53$
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iii) $x^2 + 4x + 3 < -3$

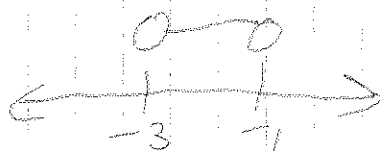
$x^2 + 4x + 3 < 0$

x	x^2	x	x
3	3x	3	3

$(x+1)(x+3) < 0$

Boundary: $x = -1, x = -3$

Test Points: $x = -5$
 $(-5)^2 + 4(-5)$
 $25 + -20$
 $5 < -3$

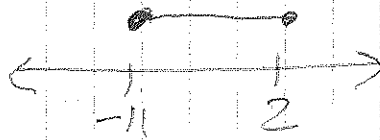


$x = -2$
 $(-2)^2 + 4(-2)$
 $4 - 8$
 $-4 < -3$

$x = 0$
 $(0)^2 + 4(0)$
 $0 < -3$

Practice:

1) $(x+11)(x-2) \leq 0$
 $x = -11, x = 2$

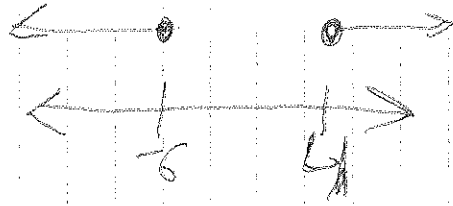


Test: $x = -100$ | $x = 0$ | $x = 100$
 $(-100+11)(-100-2)$ | $(0+11)(0-2)$ | $(100+11)(100-2)$
 $(-89)(-102)$ | $(11)(-2)$ | $111 \cdot 98$
 $\neq \leq 0$ | $-22 \leq 0$ | $\neq \leq 0$

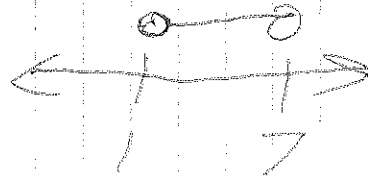
I will only draw the solutions from here on out. You should practice the test points yourself

Practice Card:

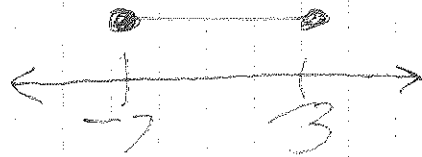
2) $(x-4)(x+6) \geq 0$
 $x=4, x=-6$



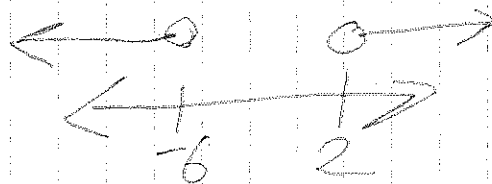
3) $(x-4)^2 + 8 < 17$
 $(x-4)^2 < 9$
 $-3 < x-4 < 3$
 $1 < x < 7$



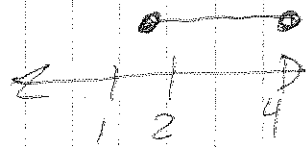
4) $3(x+5)^2 - 1 \leq 11$
 $+1 +1$
 $3(x+5)^2 \leq 12$
 $\frac{3}{3} \frac{(x+5)^2}{3} \leq \frac{12}{3}$
 $(x+5)^2 \leq 4$
 $-2 \leq x+5 \leq 2$
 $-7 \leq x \leq -3$



5) $x^2 + 4x - 12 > 0$
 $(x+6)(x-2) > 0$
 $x=-6, x=2$



6) $2x^2 - 5x < 12$
 $2x^2 - 5x - 12 < 0$



$x^2 - \frac{5}{2}x - 6 < 0$

x	$-\frac{5}{2}$	
x^2	$-\frac{5}{2}x$	-6
$-\frac{5}{4}$	$+\frac{5}{4}x$	$-\frac{25}{16}$

$(x - \frac{5}{4})^2 - 6 - \frac{25}{16} < 0$
 $(x - \frac{5}{4})^2 - \frac{121}{16} < 0$

$(x - \frac{5}{4})^2 < \frac{121}{16}$
 $-\frac{11}{4} < x - \frac{5}{4} < \frac{11}{4}$
 $\frac{6}{4} \leq x < \frac{16}{4}$
 $1.5 \leq x \leq 4$

Other Functions:

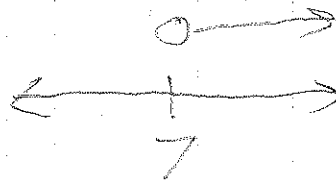
$$\text{ai) } 2\sqrt{x-3} + 1 > 5$$

$$\begin{array}{r} -1 \quad -1 \\ \hline 2\sqrt{x-3} > 4 \\ \hline 2 \quad 2 \end{array}$$

$$(\sqrt{x-3})^2 \cdot (2)^2$$

$$x-3 > 4$$

$$x > 7$$



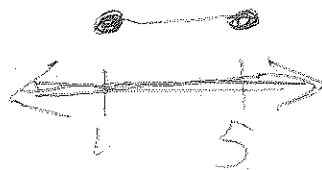
$$\text{ii) } -4|x-3| - 2 \leq -10$$

$$\begin{array}{r} +2 \quad +2 \\ \hline -4|x-3| \leq -8 \\ \hline -4 \quad -4 \end{array}$$

$$|x-3| \leq 2$$

$$-2 \leq x-3 \leq 2$$

$$1 \leq x \leq 5$$



$$\text{iii) } 2x \left(\frac{4}{x} - 2 \right) \leq \left(\frac{5}{2x} \right) \cdot 2x$$

$$8 - 4x \leq 5$$

$$\begin{array}{r} -4x \leq -3 \\ \hline -4 \quad -4 \end{array}$$

$$x \geq \frac{3}{4}$$

