CCSS Advanced Algebra 4 Solving Trigonometric Equations

1. Solve each equation below using any method (use degree mode).

a. $-12\sin(x+30)-8=-2,\ 0\le x\le 360$

$-12\sin(x+30)=6$

$\sin(x+30)=-\frac{1}{2}$

$x+30=210$

$x=180$

$x+30=330$

$x=300$

b. $2\cos(3x)-4\sqrt{2}=-3\sqrt{2},\ -180\le x\le 180$

$2\cos(3x)=1\sqrt{2}$

$\cos(3x)=\frac{\sqrt{2}}{2}$

$3x=45$

$x=15$

$3x=-45$

$x=-15$

2. Solve each equation by hand using the Unit Circle. Show your work (~~use degree mode~~).

a. $10\cos(x-10)+14=14,\ 180\le x\le 540$

$10\cos(x-10)=0$

$\cos(x-10)=0$

$x-10=270$

$x=280$

$x-10=450$

$x=460$

b. $4\sin(2x-45)=-8\sqrt{2},\ -360\le x\le -180$

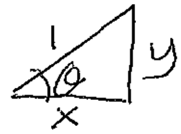
$\sin(2x-45)=-2\sqrt{2}$

Sin never reaches $-2\sqrt{2}$. The biggest value of sin is ± 1. So no solution

3. **Pythagorean Identity**: Read about the Pythagorean Identity here.

a. Write the equation of the Pythagorean Identity.

$\cos^2\theta+\sin^2\theta=1$



$x=\cos\theta$
$y=\sin\theta$

b. Why is this equation referred to as Pythagorean?

Because it's just $a^2+b^2=c^2$

c. Using the Pythagorean Identity:

i. Let θ be an angle of rotation so that $\cos(\theta)=\frac{3}{5}$ and θ is a 1st Quadrant rotation. Find $\sin(\theta)$.

$\left(\frac{3}{5}\right)^2+\sin^2\theta=1$

$\frac{9}{25}+\sin^2\theta=1$

$\sin^2\theta=\frac{16}{25}$

$\sin\theta=\frac{4}{5}$

ii. Let θ be an angle of rotation so that $\cos(\theta)=\frac{3}{5}$ and θ is a 4th Quadrant rotation. Find $\sin(\theta)$.

$\sin\theta=-\frac{4}{5}$

iii. Let θ be an angle of rotation so that $\sin(\theta)=-\frac{5}{13}$ and θ is a 2nd Quadrant rotation. Find $\cos(\theta)$.

$\left(-\frac{5}{13}\right)^2+\cos^2\theta=1$

$\frac{25}{169}+\cos^2\theta=1$

$\cos^2\theta=\frac{144}{169}$

$\cos\theta=-\frac{12}{13}$

iv. Is the point $\left(\frac{9}{41},\ -\frac{40}{41}\right)$ on the unit circle? Explain how you know.

$\left(\frac{9}{41}\right)^2+\left(-\frac{40}{41}\right)^2\overset{?}{=}1^2 \longrightarrow \frac{81}{1681}+\frac{1600}{1681}=\frac{1681}{1681}=1$. Yes.

4. The number of millions of visitors that the Washington Memorial in Washington DC gets can be modeled using the equation (in degrees) $t(x) = 2.3\sin(30x+1)+4.1$, where x represents months of the year and y = millions of tourists.

a) Determine the period of the function and explain what it means about tourists. $\frac{360}{30} = 12$. Repeats every 12 mo.

b) How many millions of tourists does the Washington Monument get in a year? Explain how you know and show your work. $4.1 \cdot 12 = 49.2$ million.

c) Which month has the most visitors? Explain how you know $x = 3$ (March)

d) In which months does the Washington Monument receive more than 6 million visitors? Explain how you know and be specific.

$2.3\sin(30x+1)+4.1 = 6$

$2.3\sin(30x+1) = 1.9$

$\sin(30x+1) = .826$

$30x+1 = 55.69$ OR $30x+1 = 124.30$

$x = 1.823$ $\quad x = 4.11$

Feb through Apr.

5. Suppose that the following table represents the average monthly ambient air temperature, in degrees Fahrenheit, in some subterranean caverns in southeast Australia for each of the twelve months in a year. We wish to model these data with a trigonometric function. (Notice that the seasons are reversed in the Southern Hemisphere, so January is in summer, and July is in winter.)

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
°F	64.04	64.22	61.88	57.92	53.60	50.36	49.10	49.82	52.34	55.22	58.10	61.52

a. Does it seem reasonable to assume that these data, if extended beyond one year, should be roughly periodic? Yes, seasons repeat.

b. What seems to be the amplitude of the data? $\frac{Max - Min}{2} = \frac{64.22 - 49.10}{2} = 7.56$

c. What seems to be the midline of the data (equation of the line through the middle of the graph)? $\frac{Max + Min}{2} = \frac{64.22 + 49.10}{2} = \frac{113.32}{2} = 56.66$

d. Would it be easier to use sine or cosine to model these data? Starts near max → cos

e. What is a reasonable approximation for the horizontal shift? 1 month

f. Write an equation for a function that could fit these data.

$$f(x) = 7.56\cos\left(\frac{2\pi}{12}(x-1)\right) + 56.66$$

6. The table below provides data for the number of daylight hours as a function of day of the year, where day 1 represents January 1.

Day of Year	0	50	100	150	175	200	250	300	350
Hours	4.0	7.9	14.9	19.9	20.5	19.5	14.0	7.1	3.6

a. Determine the period, amplitude, and midline of the function, and find an equation for a function that models the data. Max = 20.5, Min = 3.6

b. What day(s) of the year will there be 12 hours of daylight? Show or explain your work and be precise.

Amp = 8.45, Midline = 12.05, Period = 365.25

$$g(x) = 8.45\cos\left(\frac{2\pi}{365.25}x + 10\right) + 12.05$$