CCSS Advanced Algebra 4 The Special Role of Zero in Factoring

In your table groups, discuss and solve Questions 1 and 2 below:

1. Find all real solutions to the equation:
   a. $(x^2 + 5x + 6)(x^2 - 3x - 4) = 0$.
   b. $(x^2 - 9)(x^2 - 16) = 0$.

a)

	x	3
x	x^2	$3x$
2	$2x$	6

	x	-4
x	x^2	$-4x$
1	$1x$	-4

$(x+3)(x+2)(x+1)(x-4)$

$x = -3, -2, -1, 4$

b)

	x	$+3$
x	x^2	$3x$
-3	$-3x$	-9

	x	4
x	x^2	$4x$
-4	$-4x$	-16

$(x+3)(x-3)(x+4)(x-4)$

$x = -3, 3, -4, 4$

2. Suppose we know that the polynomial equation $4x^3 - 12x^2 + 3x + 5 = 0$ has three real solutions and that one of the factors of $4x^3 - 12x^2 + 3x + 5$ is $(x - 1)$. How can we find all three solutions to the given equation? Divide by $x-1$.

	$4x^2$	$-8x$	-5
x	$4x^3$	$-8x^2$	$-5x$
-1	$-4x^2$	$8x$	5

$\longrightarrow 4x^2 - 8x - 5 =$

	$2x$	-5
$2x$	$4x^2$	$-10x$
$+1$	$2x$	-5

$(x-1)(2x+1)(2x-5)$

$x = 1, \quad x = -1/2, \quad x = 5/2$

Complete Exercises 3–7 in your math notebooks:

3. Find the real zeros (also known as roots or x-intercepts) of the following polynomial functions.
   a. $f(x) = (x + 1)(x - 1)(x^2 + 1)$

      $x = -1, x = 1,$ $\quad x^2 + 1 \neq 0 \Leftarrow$ can't be solved
      $x^2 \neq -1$ b/c can't $\sqrt{-\#}$.
   b. $g(x) = (x - 4)^3(x - 2)^8$

      $x = 4, x = 2$
   c. $h(x) = (2x - 3)^5$

      $x = 3/2$
   d. $k(x) = (3x + 4)^{100}(x - 17)^4$

      $x = -4/3, \quad x = 17$

4. A Zero or Root of a Polynomial can have a **multiplicity** if the root is repeated in the function. For example, $m(x) = (x - 2)^4(x + 1)^2$ has two roots: $x = 2$ with **multiplicity** 4 and $x = -1$ with **multiplicity** 2. Find the multiplicity for each root in Question 3.

   a) Multiplicity = 1 for both
   b) $x = 4$ has mult = 3, $x = 2$ has mult = 8
   c) $x = 3/2$ has mult = 5
   d) $x = -4/3$ has mult = 100, $x = 17$ has mult = 4

5. Write a polynomial function that has the following zeros and multiplicities. What is the degree of your polynomial?

Zero	Multiplicity
2	3
– 4	1
6	6
– 8	10

$(x-2)^3(x+4)(x-6)^6(x+8)^{10}$

degree $= 3 + 1 + 6 + 10$
$= 20.$

6. Is there more than one polynomial function that has the same zeros and multiplicities as the one you found in Exercise 5? Yes. Can multiply by any constant.

Examples: $-12345(x-2)^3(x+4)(x-6)^6(x+8)^{10}$

OR $\frac{234}{317\sqrt{2}}(x-2)^3(x+4)(x-6)^6(x+8)^{10}$

7. Can you find a rule that relates the multiplicities of the zeros to the degree of the polynomial function?

Degree = Sum of multiplicities.

Problem Set

For Problems 1–4, find all solutions to the given equations.

1. $(x-3)(x+2)=0$ $x=3$, $x=-2$

2. $(x-5)(x+2)(x+3)=0$ $x=5$, $x=-2$, $x=-3$

3. $(2x-4)(x+5)=0$ $x=2$, $x=-5$

4. $(2x-2)(3x+1)(x-1)=0$ $x=1$, $x=-1/3$, $x=1$

5. Find four solutions to the equation $(x^2-1)(x^2-36)=0$.

$(x+1)(x-1)(x+6)(x-6) \longrightarrow x=1, x=-1, x=6, x=-6$

6. Find two different polynomial functions that have zeros at 1, 3, and 5 of multiplicity 1.

$5(x-1)(x-3)(x-5)$ & $13.75(x-1)(x-3)(x-5)$

7. Find two different polynomial functions that have a zero at 2 of multiplicity 5 and a zero at -4 of multiplicity 3. $573(x-2)^5(x+4)^3$ & $245.732(x-2)^5(x+4)^3$

8. If p, q, r, s are nonzero numbers, find the solutions to the equation $(px+q)(rx+s)=0$ in terms of p, q, r, s.

$x=-q/p$, $x=-s/r$

Use the identity $a^2-b^2=(a-b)(a+b)$ to solve the equations given in Problems 9-10.

9. $(x-3)(x+3)=(2x-1)(2x+1)$ $\longrightarrow$

$$x^2-9=4x^2-1$$
$$-x^2 \qquad -x^2$$
$$-9=3x^2-1$$
$$+1 \qquad +1$$
$$\frac{-8}{3}=\frac{3x^2}{3}$$
$$-\frac{8}{3}=x^2$$

No solution.

10. $(3x+2)(3x-2)=(2+3x)(2-3x)$

$$9x^2-4=4-9x^2$$
$$+9x^2 \qquad +9x^2$$
$$18x^2-4=4$$
$$+4 \quad +4$$
$$\frac{18x^2}{18}=\frac{8}{18}$$
$$\sqrt{x^2}=\sqrt{\frac{8}{18}} \rightarrow \frac{8}{18}=\frac{4}{9}$$
$$x=\frac{2}{3} \text{ or } x=-\frac{2}{3}$$