

Word Problems: Make sure to define your variables AND answer the question in words.

1. At an after season sale on winter clothes, I found a bunch of really cute hats & scarves. I decided to buy two hats and two scarves for myself to have for next year. I spent \$60. When I told my friends about the sale, they asked me to go back & get something for them. I ended up spending \$44 on one hat and two more scarves. What was the price of a single hat & a single scarf?

Let  $h$  = hat price  
Let  $s$  = scarf price

$$\begin{array}{r} 2h + 2s = 60 \\ -(1h + 2s = 44) \\ \hline 1h = 16 \end{array}$$

$$\begin{array}{r} 16 + 2s = 44 \\ -16 \\ \hline 2s = 28 \\ \frac{2s}{2} = \frac{28}{2} \\ s = 14 \end{array}$$

2. A youth group & their leaders visited Mammoth Cave. Two adults & 5 students in one van paid \$77 for the Grand Avenue Tour of the cave. Two adults & 7 students in another van paid \$95 for the same tour. Find the adult price & the student price of the tour.

Let  $a$  = adult price  
Let  $s$  = student price

$$\begin{array}{r} 2a + 5s = 77 \\ -(2a + 7s = 95) \\ \hline -2s = -18 \\ \frac{-2s}{-2} = \frac{-18}{-2} \\ s = 9 \end{array}$$

$$\begin{array}{r} 2a + 5(9) = 77 \\ 2a + 45 = 77 \\ 2a = 32 \\ \frac{2a}{2} = \frac{32}{2} \\ a = 16 \end{array}$$

3. Friends from the math department often pick up lunch for each other. When it was Mr. Linnenbringer's turn to make the food run, he bought 5 sandwiches and 3 bags of chips. He spent \$29.50. When Mr. Willms went, he got 4 sandwiches and 4 bags of chips for \$26. How much does a sandwich cost? How about a bag of chips?

Let  $s$  = sandwich price  
Let  $c$  = chip price

$$\begin{array}{l} 4 \cdot (5s + 3c = 29.50) \cdot 4 \rightarrow 20s + 12c = 118 \\ 5 \cdot (4s + 4c = 26) \cdot 5 \rightarrow 20s + 20c = 130 \end{array}$$

$$\begin{array}{r} 20s + 12c = 118 \\ -(20s + 20c = 130) \\ \hline -8c = -12 \\ \frac{-8c}{-8} = \frac{-12}{-8} \\ c = 1.50 \end{array}$$

$$\begin{array}{l} 5s + 3(1.50) = 29.50 \\ 5s + 4.50 = 29.50 \\ 5s = 25 \\ s = 5 \end{array}$$

4. A website allows users to download individual songs or an entire album. All individual songs cost the same to download, and all albums cost the same to download. Ryan pays \$14.94 to download 5 individual songs and 1 album. Seth pays \$22.95 to download 3 individual songs and 2 albums. How much does the website charge to download a song? How about an entire album?

Let  $s$  = song price  
Let  $a$  = album price

$$\begin{array}{r} 2(5s + 1a = 14.94) \cdot 2 \\ (3s + 2a = 22.95) \\ \hline 2(10s + 2a = 29.88) \cdot 2 \\ \hline -7s = -6.93 \\ \frac{-7s}{-7} = \frac{-6.93}{-7} \\ s = 0.99 \end{array}$$

$$\begin{array}{r} 5(0.99) + 1a = 14.94 \\ 4.95 + a = 14.94 \\ a = 9.99 \end{array}$$

Pure equations: Use elimination or substitution to solve for x AND y. Use scratch paper if necessary.

**C Level:**

Solve each system of equations. Check your solution:

1.  $\begin{cases} 3x + 2y = -18 \\ 4x - 5y = -70 \end{cases} \rightarrow \begin{cases} 12x + 8y = -72 \\ 3(4x - 5y) = -210 \end{cases} \rightarrow \begin{cases} 12x + 8y = -72 \\ 12x - 15y = -210 \end{cases}$   
 $23y = 138 \rightarrow y = 6$   
 $3x + 2(6) = -18 \rightarrow 3x + 12 = -18 \rightarrow 3x = -30 \rightarrow x = -10$

2.  $\begin{cases} y = x^2 - 3x + 11 \\ x^2 - y = 10 \end{cases} \rightarrow x^2 - (x^2 - 3x + 11) = 10 \rightarrow 3x - 11 = 10 \rightarrow 3x = 21 \rightarrow x = 7$   
 $y = 7^2 - 3(7) + 11 = 49 - 21 + 11 = 39$

3.  $\begin{cases} 4x^2 - y^2 + 3x = 7 \\ 4x^2 + y^2 + x = 25 \end{cases} \rightarrow \begin{cases} 4x^2 - y^2 + 3x = 7 \\ 4x^2 + y^2 + x = 25 \end{cases}$   
 $4x = 32 \rightarrow x = 8$   
 $4(8)^2 - y^2 + 3(8) = 7 \rightarrow 4(64) - y^2 + 24 = 7 \rightarrow 256 - y^2 + 24 = 7 \rightarrow 280 - y^2 = 7 \rightarrow -y^2 = -273 \rightarrow y^2 = 273 \rightarrow y = \pm\sqrt{273}$

4.  $\begin{cases} y = |x - 6| \\ y = 2x \end{cases} \rightarrow |x - 6| = 2x$   
 $x - 6 = 2x \rightarrow -6 = x \rightarrow x = -6$   
 $y = 2(-6) = -12$

$x - 6 = -2x \rightarrow 3x = 6 \rightarrow x = 2$   
 $y = 2(2) = 4$

5.  $\begin{cases} 3(x - 2)^2 + y = 28 \\ y = 1 \end{cases} \rightarrow 3(x - 2)^2 + 1 = 28 \rightarrow 3(x - 2)^2 = 27 \rightarrow (x - 2)^2 = 9$   
 $x - 2 = 3 \rightarrow x = 5$   
 $x - 2 = -3 \rightarrow x = -1$

6.  $\begin{cases} \sqrt{2x + 3} + 8 = y \\ y = 10 \end{cases} \rightarrow \sqrt{2x + 3} + 8 = 10 \rightarrow \sqrt{2x + 3} = 2$   
 $\sqrt{2x + 11} = 10$   
 $2x + 11 = 100$   
 $2x = 89 \rightarrow x = 44.5$

**A/B Level:**

Solve each system of equations. Check your solution:

7.  $\begin{cases} x^2 + y^2 = 16 \\ y = \sqrt{2x - 32} \end{cases} \rightarrow x^2 + (\sqrt{2x - 32})^2 = 16$   
 $x^2 + 2x - 32 = 16 \rightarrow x^2 + 2x - 48 = 0$   
 $(x + 8)(x - 6) = 0 \rightarrow x = -8 \text{ or } x = 6$   
 $y = \sqrt{2(6) - 32} = \sqrt{-20}$   
 $y$  does not exist

8.  $\begin{cases} y = (x + 1)(x - 3) \\ x^2 + x - y = 12 \end{cases} \rightarrow x^2 + x - (x + 1)(x - 3) = 12$   
 $x^2 + x - (x^2 - 2x - 3) = 12 \rightarrow x^2 + x - x^2 + 2x + 3 = 12 \rightarrow 3x + 3 = 12 \rightarrow 3x = 9 \rightarrow x = 3$   
 $y = (3 + 1)(3 - 3) = 0$

9.  $\begin{cases} 2|x - 3| + y^2 = 7 \\ |x - 3| - y^2 + 4 = 3 \end{cases} \rightarrow \begin{cases} 2|x - 3| + y^2 = 7 \\ |x - 3| - y^2 + 4 = 3 \end{cases}$   
 $3|x - 3| + 4 = 10 \rightarrow 3|x - 3| = 6 \rightarrow |x - 3| = 2$   
 $x - 3 = 2 \rightarrow x = 5$   
 $x - 3 = -2 \rightarrow x = 1$   
 $2(5 - 3) + y^2 = 7 \rightarrow 4 + y^2 = 7 \rightarrow y^2 = 3 \rightarrow y = \pm\sqrt{3}$

Challenge: Use a graphing calculator or desmos if you get stuck.

10. During the Great Water Balloon Contest (held at the Cleveland football field), Phaëdrus placed his catapult on the 10 yard line. His water balloon followed the equation  $p(x) = -(x - 10)(x - 20)$ , where  $x$  = distance along the ground in yards and  $y$  = height in the air in yards.

At exactly the same time, Mallach launched his water balloon which followed the equation  $m(x) = -(x - 20)^2 + 36$ .

Did Mallach and Phaëdrus's water balloons collide in flight? Use systems of equations to answer the question and explain specifically what the solution to the system of equations tells you about this situation.

$$-(x - 10)(x - 20) = -(x - 20)^2 + 36$$

$$-(x^2 - 30x + 200) = -(x^2 - 40x + 400) + 36$$

$$-x^2 + 30x - 200 = -x^2 + 40x - 400 + 36$$

$$30x - 200 = 40x - 364$$

$$164 = 10x$$

$$16.4 = x$$

Yes. They collide at the 16.4 yard line. At a height of  $-(16.4 - 10)(16.4 - 20) = -(6.4)(-3.6) = 23.04$  yards