CCSS Algebra 3 Translating Exponents to Logarithms

Remember that logarithms are just an invention for solving equations where the exponents are variables. Just like roots are the inverses of powers (squares, cubes, fourth powers), logarithms are the inverses of exponents.

Power	Root	Exponent	Logarithm
$X^5 = 32$	$X = \sqrt[5]{32} = 2$	$7^x = 343$	$X = \log_7 343 = 3$

1. Convert each exponential equation into a logarithmic equation and use a calculator to solve for x:

a. $3^x = 40$

$\log_3 \quad \log_3$

$x = \log_3 40$

b. $12^{2x} = 1728$

$\log_{12} \quad \log_{12}$

$2x = \log_{12} 1728$

c. $4^x + 5 = 40$

$-5 \quad -5$

$\log_4 4^x = 35 \quad \log_4$

$x = \log_4 35$

d. $10(2)^{x+4} = 640$

$10 \quad 10$

$\log_2 2^{x+4} = 64 \quad \log_2$

$x + 4 = \log_2 64$

e. $5(3)^x - 7 = 42$

$+7 \quad +7$

$\frac{5(3)^x}{5} = \frac{49}{5}$

$\log_3 3^x = 9.8 \quad \log_3 \rightarrow x = \log_3 9.8$

2. Convert each logarithmic equation into an exponential equation and use a calculator to solve for x:

a. $\cancel{\log_8}(x) = 2$

$8 \quad 8$

$x = 8^2$

b. $\cancel{\log_3}(x+5) = 4$

$3 \quad 3$

$x + 5 = 3^4$

c. $\log_2(x) + 3 = 2$

$-3 \quad -3$

$\cancel{\log_2}(x) = -1$

$2 \quad 2$

d. $3\log_{10}(x) = -6$

$\frac{\quad}{3} \quad \frac{\quad}{3}$

$\cancel{\log_{10}} x = -2$

$10 \quad 10$

$x = 10^{-2}$

e. $2\log_6(x) - 1 = 5$

$+1 \quad +1$

$\frac{2\log_6 x}{2} = \frac{6}{2}$

$\cancel{\log_6} x = 3$

$6 \quad 6$

$x = 6^3$

3. Find the inverse of each function below:

a. $f(x) = 3(2)^x - 1$

$+1 \quad +1$

$\frac{y+1}{3} = \frac{3(2)^x}{3}$

$\log_2 \frac{y+1}{3} = 2^x \quad \log_2$

$\log_2 \frac{y+1}{3} = x$

b. $g(x) = 4^{x-3} + 5$

$-5 \quad -5$

$\log_4 y - 5 = 4^{x-3} \quad \log_4$

$\log_4(y-5) = x - 3$

$\log_4(y-5) + 3 = x$

c. $h(x) = \log_3(x-5) + 2$

$-2 \quad -2$

$3 \quad y - 2 = \cancel{\log_3}(x-5)$

3

$3^{y-2} = x - 5$

$+5 \quad +5$

$3^{y-2} + 5 = x$

d. $j(x) = 5\log_{10}(2x-1)$

$\frac{\quad}{5} \quad \frac{\quad}{5}$

$10 \quad \frac{y}{5} = \cancel{\log_{10}}(2x-1)$

10

$10^{y/5} = 2x - 1$

$+1 \quad +1$

$\frac{10^{y/5} + 1}{2} = \frac{2x}{2}$

$\frac{10^{y/5}+1}{2} = x$

4. Exponents have certain properties that you can use to simplify expressions and equations. Logarithms, because they are the inverse of exponentials, have related properties. Remember that inverses switch inputs and outputs, so the rules will be related, but not exactly the same. Use the example in the first line to fill in the table.

Exponent Law	Logarithmic Law
$3^x \cdot 3^y = 3^{x+y}$	$log_3 x + log_3 y = log_3(x \cdot y)$
$\frac{7^x}{7^y} = 7^{x-y}$	$\log_7(x/y) = \log_7 x - \log_7 y$
$(3^x)^4 = 3^{4 \cdot x}$	$log_4 x^3 = 3 log_4 x$

5. Simplify each expression using the exponential or logarithmic laws:

a. $5^3 \cdot 5^2 = 5^5 = 3125$

b. $log_3 9 + log_3 27 = \log_3 243 = 5$

c. $\frac{12^3}{12^5} = 12^{-2} = \frac{1}{144}$

d. $log_7 49 - log_7 343 = \log_7\left(\frac{49}{343}\right) = \log_7(1/7) = -1$

e. $(5^3)^2 = 5^6$

f. $log_6 x^3 = 3\log_6 x$

6. The world population is growing exponentially. From 1960 to 1980, the world population increased from 3,000,000,000 to 4,400,000,000. This can be modeled by the equation $w(t) = 3(1.467)^{t/20}$, where w(t) represents the population of the world (in billions of people) and where t represents the time (in years).

a. What is w(0)? What does this represent about the world population? Did you need a calculator to answer this?

$w(0) = 3$. 3 billion people were alive in 1960.

b. What is w(20)? What does this represent about the world population? Did you need a calculator to answer this?

$w(20) = 4.4$. 4.4 billion people were alive in 1980.

c. What is w(58)? What does this represent about the world population? Did you need a calculator to answer this?

$w(58) = 3(1.467)^{58/20} = 9.115$

9.115 billion people are alive in 2018.

d. Find the inverse of $w(t)$ and label your new equation $w^{-1}(t)$.

$w^{-1}(t) = 20 \cdot \log_{1.467}(t/3)$