

Remember that logarithms are just an invention for solving equations where the exponents are variables. Just like roots are the inverses of powers (squares, cubes, fourth powers), logarithms are the inverses of exponents.

Power	Root	Exponent	Logarithm
$x^5 = 32$	$x = \sqrt[5]{32} = 2$	$7^x = 343$	$x = \log_7 343 = 3$

1. Convert each exponential equation into a logarithmic equation and use a calculator to solve for x:

a. $3^x = 40$

$$\begin{aligned} x &= \log_3 40 \\ x &= 3.36 \end{aligned}$$

b. $12^{2x} = 1728$

$$\begin{aligned} \log_{12} 1728 &= 2x \\ 3 &= 2x \\ 1.5 &= x \end{aligned}$$

c. $4^x + 5 = 40$

$$\begin{aligned} 4^x &= 35 \\ \log_4 35 &= x \\ 2.56 &= x \end{aligned}$$

d. $10(2)^{x+4} = 640$

$$\begin{aligned} 2^{x+4} &= 64 \\ \log_2 64 &= x+4 \\ 6 &= x+4, x=2 \end{aligned}$$

e. $5(3)^x - 7 = 42$

$$\begin{aligned} 5(3)^x &= 49 \\ 3^x &= 9.8 \\ x &= \log_3 9.8 = 2.08 \end{aligned}$$

2. Convert each logarithmic equation into an exponential equation and use a calculator to solve for x:

a. $\log_8(x) = 2$

$$\begin{aligned} 8^2 &= x \\ 64 &= x \end{aligned}$$

b. $\log_3(x+5) = 4$

$$\begin{aligned} 3^4 &= x+5 \\ 81 &= x+5 \\ 76 &= x \end{aligned}$$

c. $\log_2(x) + 3 = 2$

$$\begin{aligned} \log_2 x &= -1 \\ 2^{-1} &= x \\ \frac{1}{2} &= x \end{aligned}$$

d. $3 \log_{10}(x) = -6$

$$\begin{aligned} \log_{10}(x) &= -2 \\ 10^{-2} &= x = \frac{1}{100} \end{aligned}$$

e. $2 \log_6(x) - 1 = 5$

$$\begin{aligned} 2 \log_6 x &= 6 \\ \log_6 x &= 3 \\ 6^3 &= x = 216 \end{aligned}$$

3. Find the inverse of each function below:

a. $f(x) = 3(2)^x - 1$

$$\begin{aligned} 2^x &\rightarrow 1 \cdot 3 \rightarrow +1 \rightarrow \log_2(\frac{x+1}{3}) \\ -1 &\rightarrow \log_2 x \end{aligned}$$

b. $g(x) = 4^{x-3} + 5$

$$\begin{aligned} 4^x &\rightarrow -3 \rightarrow 1 \rightarrow -5 \rightarrow \log_4(x-5) + 3 \\ +5 &\rightarrow +3 \end{aligned}$$

c. $h(x) = \log_3(x-5) + 2$

$$\begin{aligned} -5 &\rightarrow 1 \rightarrow -2 \rightarrow 3^x \rightarrow (x-2) \rightarrow +5 \\ \log_3 &\rightarrow +2 \end{aligned}$$

d. $j(x) = 5 \log_{10}(2x-1)$

$$\begin{aligned} 2 &\rightarrow -1 \rightarrow \frac{1}{2} \rightarrow 10^x \rightarrow \frac{(x+1)}{2} \\ -5 &\rightarrow -2 \end{aligned}$$