Remember: Exponential Functions are written $y = b(m)^x$, where b is the initial value, and m is the change.

1. **Zero Exponents**

a. Since 2000, college tuition has been growing substantially. For example, the cost of tuition at the University of Oregon since 2000 is shown in the table below. Is the cost of tuition a LINEAR pattern or an EXPONENTIAL pattern? How do you know? Exponential b/c tuition is multiplied

Year	2000	2001	2002	2003	2004	2005
Tuition ($)	3,800	4,026	4,265.5	4,519.2	4,788	5,072.8

b. Kianna believes that the equation for the table should be $y = 3800(226)^x$. Why is she not correct?

Because 226 is not the multiplier. She thought linearly.

c. What do you do to figure out the **growth factor** for an exponential function?

Divide. later ÷ Earlier, $\frac{y_2}{y_1}$.

d. Write an exponential function to model the price of tuition at U of O since 2000, where x = years since 2000 and y = tuition for Oregon residents. Remember to use $y = b(m)^x$

$y = 3800(1.06)^x$

e. Check that your function works by using at least 2 (x,y) points from your table above.

$4026 = 3800(1.06)^1$ $\quad$ $4265.5 = 3800(1.06)^2$

f. Saul chose the point (0, 3800) to check if his function works. The function he wrote down was $y = 3800(1.06)^x$. Plug in $x = 0$ and $y = 3800$ into the equation and show your work and answer below.

$3800 = 3800(1.06)^0$

g. It seems weird that when you plug in the point (0, 3800), you get the equation $3800 = 3800(1.06)^0$. Divide both sides of your equation by 3800. Write your answer below.

$\frac{3800}{3800} = \frac{3800(1.06)^0}{3800} \longrightarrow 1 = (1.06)^0$

h. On problem (g), you should have written the equation $1 = (1.06)^0$, because you divided both sides by 3800 (and $\frac{3800}{3800} = 1$). Write a conjecture about what **zero exponents** do to numbers.

Anything to the zero is one.

Zero Exponents: Anything to the zero power is one. $x^0 = 1$

2. **Negative Exponents**

a. Remember that the function for the cost of tuition is $y = 3800(1.06)^x$. How can you use your function to tell the tuition in the year 2001? Plug in $x=1$.

$y = 3800(1.06)^1 = 4028$

b. What if you wanted to know the tuition in the year 2002? 2050? 2100?

$3800(1.06)^2 = 4269.68$, $3800(1.06)^{50} = 69996.59$

$3800(1.06)^{100} = 1289347.92$

c. How could Saul use his function to determine the cost of tuition the year BEFORE he went to college? In other words, what was the tuition in the year 1999? Plug in $x=-1$

$3800(1.06)^{-1} = 3584.91$

d. What if you wanted to know the tuition in the year 1996? 1990? 1950?

$3800(1.06)^{-4} = 3009.96$ | $3800(1.06)^{-10} = 2121.90$

$3800(1.06)^{-50} = 206.30$

e. Write a sentence about how you use an exponential function to predict values in the future and in the past. Your sentence should use the words POSITIVE, NEGATIVE, FUTURE, and PAST.

Positive values for future
Negative values for past.

f. When Mr. Maurer interprets his function $y = 3800(1.06)^x$, he sees it as "I started out paying \$3800, and then I paid 1.06 times as much each year (a 6% increase)." Each year in college, the cost of tuition was multiplied by 1.06. What could you do to work backwards? In other words, what happens to the cost of tuition as you go back in time?

Divide by 1.06

g. Fill in the table below for the cost of tuition. Remember that $x = 0$ means 2000.

x=year	-3	-2	-1	0	1	2
y=tuition	3190.6	3382	3584.9	3800	4028	4269.7

Negative Exponents: $x^{-a} = \frac{1}{x^a}$ **. A negative exponent turns your base into a fraction. Negative exponents divide.**

3. **Fraction Exponents and Roots**

a. Many students graduate college in 4 years, so it is useful to think about the tuition cost over a 4-year period. The cost of tuition at the University of Oregon can be represented by the function $t(x) = 3800 \cdot (1.26)^{\frac{x}{4}}$, where x = years since 2000 and y = tuition for Oregon residents. Compare this function to $y = 3800 \cdot (1.06)^{x}$. How are they similar and different?

Both start with 3800. The t(x) function has a bigger growth factor [(1.26) vs (1.06)] but has a fraction exponent [(x/4) vs (x)]

b. Multiply $3800(1.26)$. What year's tuition did you just figure out?

$4788 → 2004

c. Multiply $3800(1.06)$. What year's tuition did you just figure out?

$4028 → 2001

d. When Mr. Maurer interprets the function $t(x) = 3800 \cdot (1.26)^{\frac{x}{4}}$, he sees it as "Tuition started out costing $3800 and then was multiplied by 1.26 every 4 years (a 26% increase)." How could he use his function to figure out the growth factor for one year?

Plug in x=1, then divide.

$3800(1.26)^{1/4} = 4026$

$\frac{4026}{3800} = 1.06$

e. For the tuition function, the growth factor of 1.26 takes 4 years to multiply. This means that each year, tuition is multiplying by a smaller growth factor, and that 4 of those multiplications result in 1.26. In other words, if m is the ONE-YEAR growth factor, then $m \cdot m \cdot m \cdot m = 1.26$. Rewrite that equation using exponents.

$m^4 = 1.26$

f. Solving equations like $x^2 = 9$ requires a new mathematical tool, called a **square root** ($\sqrt{\ }$). The square root of a number is the value that, multiplied by itself, equals the number. For example, $\sqrt{9} = 3$, $\sqrt{25} = 5$, and $\sqrt{10000} = 100$. Find the value of each square root below:

$\sqrt{36} =$ 6 $\quad \sqrt{100} =$ 10 $\quad \sqrt{400} =$ 20 $\quad \sqrt{1} =$ 1 $\quad \sqrt{0} =$ 0

g. It's also possible to solve equations with higher powers, such as the equation $x^3 = 8$. Instead of using a square root, you use the **third root** (AKA the "cube root"). Similar to a square root, a cube root is the value that, multiplied by itself **three times**, equals the number. For example, $\sqrt[3]{8} = 2$ because $2 \cdot 2 \cdot 2 = 8$. $\sqrt[3]{1000} = 10$ because $10 \cdot 10 \cdot 10 = 1000$. Find the value of each cube root below:

$\sqrt[3]{27} =$ 3 $\quad \sqrt[3]{125} =$ 5 $\quad \sqrt[3]{343} =$ 7 $\quad \sqrt[3]{1} =$ 1 $\quad \sqrt[3]{0} =$ 0

h. In fact, there is nothing special about the 2nd and 3rd power - every power has a root. Use your answers from (f) and (g) to solve each root below. The answers are all (small) whole numbers.

$\sqrt[4]{16} = 2$ $\qquad$ $\sqrt[6]{15625} = 5$ $\qquad$ $\sqrt[7]{2187} = 3$ $\qquad$ $\sqrt[11]{1} = 1$ $\qquad$ $\sqrt[13]{0} = 0$

i. In problem (e), you should have written the equation $m^4 = 1.26$. Solve that equation for m using a root. Ask Maurer for calculator help if you need it.

$$\sqrt[4]{m^4} = \sqrt[4]{1.26}$$

$$m = \sqrt[4]{1.26} = 1.06$$

j. What does it mean to raise a number to a fractional exponent. Consider the examples below:

$9^{\frac{1}{2}} = (3 \cdot 3)^{\frac{1}{2}} = 3$ $\qquad$ $16^{\frac{1}{4}} = (2 \cdot 2 \cdot 2 \cdot 2)^{\frac{1}{4}} = 2$ $\qquad$ $125^{\frac{1}{3}} = (5 \cdot 5 \cdot 5)^{\frac{1}{3}} = 5$

So what does $\quad 100^{\frac{1}{2}} =$ <u>10</u> ? $\quad 8^{\frac{1}{3}} =$ <u>2</u> ? $\quad 256^{\frac{1}{4}} =$ <u>4</u> ?

k. A fractional exponent is equivalent to taking a root. For example, $a^{\frac{1}{2}} = \sqrt{a}$ and $b^{\frac{1}{3}} = \sqrt[3]{b}$. Given part ~~k~~ j above, explain why this makes sense.

Because $\sqrt{a} \cdot \sqrt{a} = a$, & $a^{1/2} \cdot a^{1/2} = a$,

and $\sqrt[3]{b} \cdot \sqrt[3]{b} \cdot \sqrt[3]{b} = b$, & $b^{1/3} \cdot b^{1/3} \cdot b^{1/3} = b$

l. The **POWER of a POWER rule** says that $(x^a)^b = x^{a \cdot b}$. In other words, if you have a power to a power, multiply the two exponents. Use the POWER of a POWER rule and the definition of a root to prove that a fractional exponent is equivalent to a root.

Definition of a root: If $r = \sqrt[n]{x}$, then $r^n = x$.

Claim: $\sqrt[n]{x} = x^{1/n}$. $\qquad$ Proof: $(\sqrt[n]{x})^n = x$

$(x^{1/n})^n = x^{\frac{1}{n} \cdot n} = x^1$

THESE ARE EQUAL

Practice: Use the properties of exponents to rewrite each expression. Calculate the value, if you can.

1. $x^0 = 1$
2. $x^{-5} = 1/x^5$
3. $x^2 \cdot x^5 = x^7$
4. $x^{1/2} = \sqrt{x}$
5. $\sqrt[3]{x} = x^{1/3}$
6. $27^{1/3} = 3$
7. $4^{1/2} = 2$
8. $1^{1/20} = 1$