CCSS Advanced Algebra 4 Understanding Exponents Name ________________

1. **Zero Exponents**

a. Since 2000, college tuition has been growing substantially. For example, the cost of tuition at the University of Oregon since 2000 can be represented by the function $t(x) = 3800 \cdot 1.26^{\frac{x}{4}}$, where x = years since 2000 and y = tuition for Oregon residents.

i. What does the $1.26^{\frac{x}{4}}$ in the equation tell you about the cost of tuition? Be specific and complete. Tuition increases 26% every 4 years

ii. Evaluate $t(0)$. What does $t(0)$ mean about the cost of tuition?

$t(0) = 3800 \cdot 1.26^{0/4} = 3800$

Tuition was $3800 in the year 2000.

b. For any exponential equation, $f(x) = a \cdot b^x$, explain why $f(0) = a$.

Because a is the initial value.

c. Given part (b), it must be true that $a = a \cdot b^0$ for any exponential equation. Why does this mean that $b^0 = 1$ for any value b? Explain thoroughly.

Because if $\frac{a}{a} = \frac{a \cdot b^0}{a}$, $1 = b^0$. The only number you can multiply by that leaves your number unchanged is 1.

2. **Negative Exponents**

a. Mr. Slusher started college in 1984. Use the equation from 1a above to determine tuition in 1984 (recall that x = years since 2000) at the U of O.

$t(-16) = 3800 \cdot 1.26^{-16/4} = 1507.65$

b. What exponent did you use in 2a to go back to 1984? What effect did a negative exponent have on the year 2000 tuition?

I used −16, which made the tuition decrease.
Negative exponents divide.

c. Without using a calculator, predict what the value of $10 \cdot 2^{-1} =$ 5. Why do you think it will be that value? Because $10 \cdot 2^1 = 20$
$10 \cdot 2^0 = 10$, so $10 \cdot 2^{-1} = 10 \div 2 = 5$

d. Recall that exponents are human inventions to provide a shortcut for repeated multiplication. For example, $10 \cdot 2^3 = 10 \cdot 2 \cdot 2 \cdot 2$ and $10 \cdot 2^2 = 10 \cdot 2 \cdot 2$ and $10 \cdot 2^1 = 10 \cdot 2$ and $10 \cdot 2^0 = 10$. Given this pattern, what do you think $10 \cdot 2^{-1} =$ 5? What about $10 \cdot 2^{-2} =$ 2.5?

$10 \cdot 2^{-1} = \frac{10}{2}$ $10 \cdot 2^{-2} = \frac{10}{2^2} = \frac{10}{4}$

e. Use this to explain why $y = 2^{-x}$ is equivalent to $y = (\frac{1}{2})^x$. (Or equivalent to $y = \frac{1}{2^x}$)

Because negatives are the opposite of positives, ~~all~~ negative exponents mean division

3. **Fraction Exponents and Roots**

a. Use the equation from question 1 to determine the U of O tuition in 2001. In other words, evaluate $t(1) = 3800 \cdot 1.26^{\frac{1}{4}}$.

4026.02

b. In question 1ai, you should have made a statement equivalent to "$1.26^{\frac{x}{4}}$ means that tuition grew by 26% every 4 years." How could you use the answer to 3a to determine the <u>one-year</u> growth rate of tuition?

$\frac{4026.02}{3800} = 1.059 \rightarrow 5.9\%$ growth

c. What does it mean to raise a number to a fractional exponent. Consider the examples below:

$9^{\frac{1}{2}} = (3 \cdot 3)^{\frac{1}{2}} = 3$ $\quad 16^{\frac{1}{4}} = (2 \cdot 2 \cdot 2 \cdot 2)^{\frac{1}{4}} = 2$ $\quad 125^{\frac{1}{3}} = (5 \cdot 5 \cdot 5)^{\frac{1}{3}} = 5$

So what does $100^{\frac{1}{2}} =$ 10 ? $\quad 8^{\frac{1}{3}} =$ 2 ?

$(10 \cdot 10) = 100$ $\quad (2 \cdot 2 \cdot 2) = 8$

d. A fractional exponent is equivalent to taking a root. For example, $a^{\frac{1}{2}} = \sqrt{a}$ and $b^{\frac{1}{3}} = \sqrt[3]{b}$. Given part c above, explain why this makes sense.

Because a root is like multiplying by part of a number. Squar root is half-way multiplying, cube root is 1/3 multiplying, etc