

1. Good news! The vending machine in the cafeteria has broken so that you can get a drink without putting in any money. Levi runs down to the vending machine, presses Button 1 and gets a Vitamin Water. He presses Button 1 again and gets another Vitamin Water. He presses Button 2 and gets a Gatorade; when he presses Button 2 again, he gets a Vitamin Water.

- a. We say that an operation is a relation when a distinct input leads to an output. What are the inputs and outputs for the vending machine?
*Input = Button
 output = Drink*
- b. When an relation (machine or otherwise) is operating consistently, it is called a function. Is the vending machine operating as a function for Levi? Explain why or why not.

No. Button 2 is inconsistent.

- c. More formally, functions are relations in which a given input always results in the only one output. Explain what this formal definition means for the vending machine. Under what conditions would the vending machine be a function? (This would be a good time to define **function** in your notes).

Function: Each button gives one drink consistently.

- d. When operating normally, the vending machine should follow the table below:

Button	1	2	3	4	5
Drink	Vitamin Water	Gatorade	Vitamin Water	Gatorade	Orange Juice

Is the vending machine normally a function? Explain why or why not. What is the domain and range for the vending machine?

*Yes. Each button gives a single type of drink. D = Drink
 R = Button*

- e. Recall that the inverse of a relation reverses the input and outputs. What would the table look like for the inverse of the normally operating vending machine?

<i>Drink</i>	<i>VW</i>	<i>G</i>	<i>VW</i>	<i>G</i>	<i>OJ</i>
<i>Button</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>

- f. What is the domain and range of the inverse of the vending machine? How does it compare to the domain and range of the first table? (This would be a good time to put information about the domain and range of inverse functions in your notes).

*D: Drink
 R: Button*

Domain & Range are switched.

- g. Is the inverse of the vending machine a function? Explain why or why not.

No. VW & G are inconsistent because they came from 2 different buttons.

Mathematical Functions:

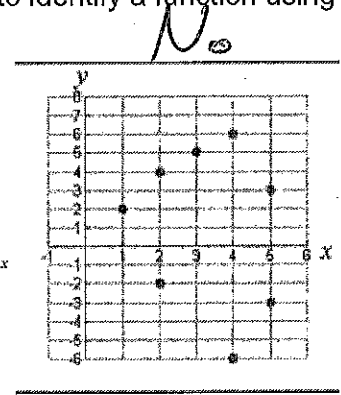
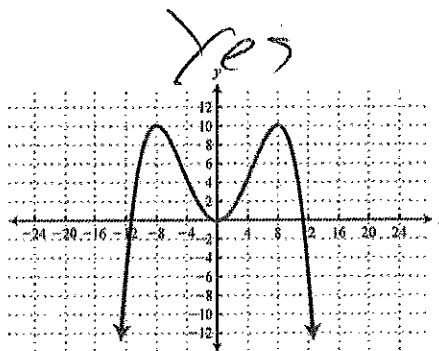
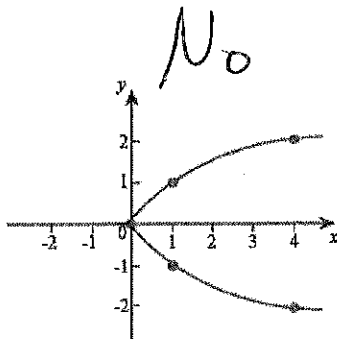
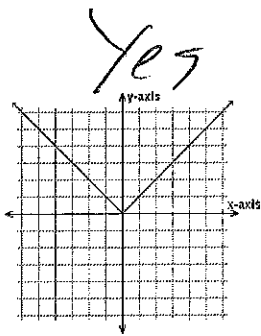
2. a. Complete the table below for the function $g(x) = (x - 1)^3 + 2$

x	-1	0	1	2	3
y	-6	1	2	3	10

b. Based on the table, is $g(x)$ a function? *Yes*

c. Go to [desmos.com](https://www.desmos.com) and graph $g(x)$. Can you locate any values of x (inputs) that have more than one output (y)? *No*

d. Read [Vertical Line Test \(all 3 slides\)](#). Which of the relations below are functions? Justify your answer. (This would be a good time to put information about how to identify a function using a graph in your notes). *No*



f. Using [desmos.com](https://www.desmos.com), graph the relation $x^2 + y = 4$.

i. Is this relation a function? Explain why or why not. *Yes, vertical line test*

ii. By switching the input (x) and output (y), graph the inverse of this relation. Is it a function? Explain why or why not. *No. Vertical line test*

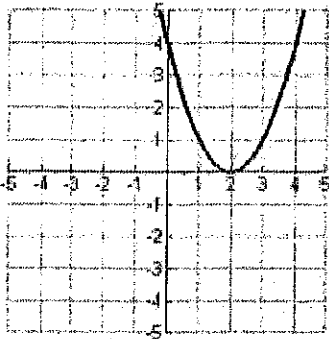
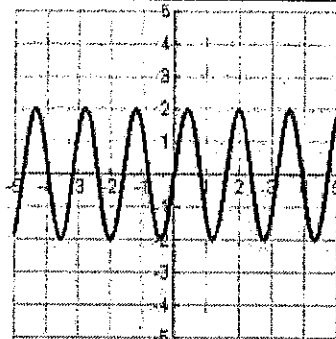
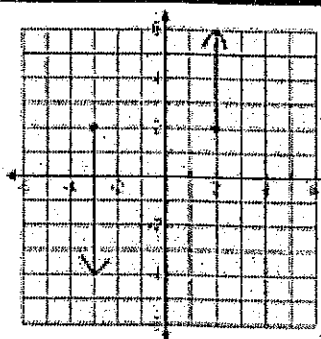
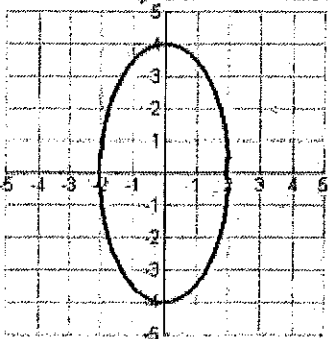
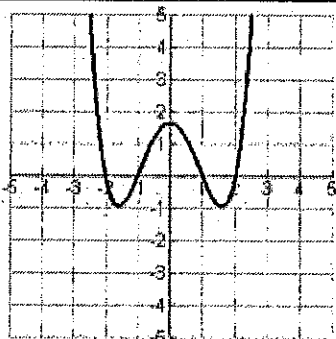
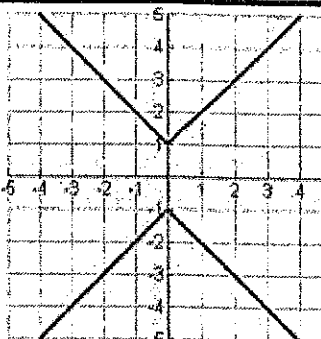
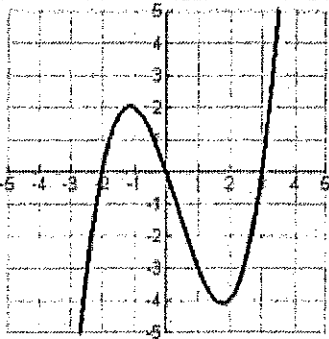
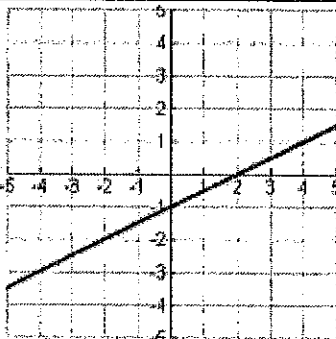
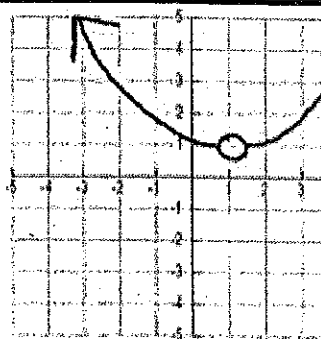
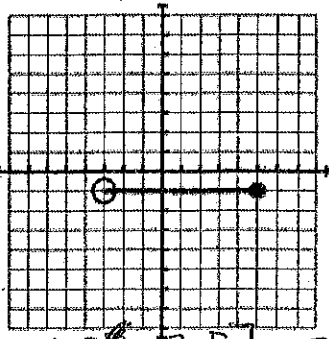
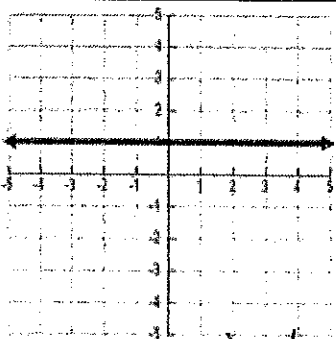
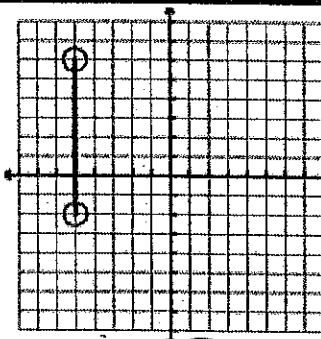
iii. Repeat parts i and ii for each relation below:

- $x + y = 7$ *Yes*
- $y = (x - 1)^3 + 2$ *Yes*
- $y = 2|x + 4|$ *Yes*
- $0.25x^3 - y = 1$ *Yes*
- $4x^2 + y^2 = 25$ *No*

f. Under what conditions will both a relation and its inverse be functions? When will one be a function and one not be a function? Are there situations in which both will not be functions? Be specific.

Both are functions if every horizontal & vertical line intersects only once. Both are not functions if any horizontal or vertical line intersects more than once.

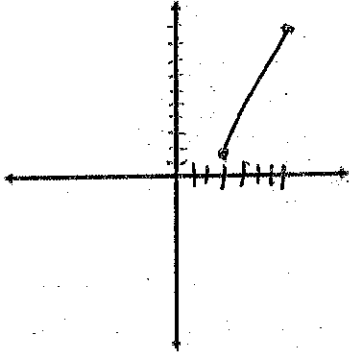
1. Find the domain and range for each graph. Then determine if the graph is a function.

 <p>D: $(-\infty, \infty)$ R: $[-1, \infty)$ Function: <u>Yes</u></p>	 <p>D: $(-\infty, \infty)$ R: $[-2, 2]$ Function: <u>Yes</u></p>	 <p>D: $[-3, 2]$ R: $(-\infty, \infty)$ Function: <u>No</u></p>
 <p>D: $[-2, 2]$ R: $[-4, 4]$ Function: <u>No</u></p>	 <p>D: $(-\infty, \infty)$ R: $[-1, \infty)$ Function: <u>Yes</u></p>	 <p>D: $(-\infty, \infty)$ R: $(-\infty, 1] \cup [1, \infty)$ Function: <u>No</u></p>
 <p>D: $(-\infty, \infty)$ R: $(-\infty, \infty)$ Function: <u>Yes</u></p>	 <p>D: $(-\infty, \infty)$ R: $(-\infty, \infty)$ Function: <u>Yes</u></p>	 <p>D: $(-\infty, 1) \cup (1, \infty)$ R: $(1, \infty)$ Function: <u>Yes</u></p>
 <p>D: $[-3, 5]$ R: $[-1, 1]$ F: <u>Yes</u></p>	 <p>D: $(-\infty, \infty)$ R: $[1, 1]$ F: <u>Yes</u></p>	 <p>D: $[-2, -2]$ R: $(-2, 6)$ F: <u>No</u></p>

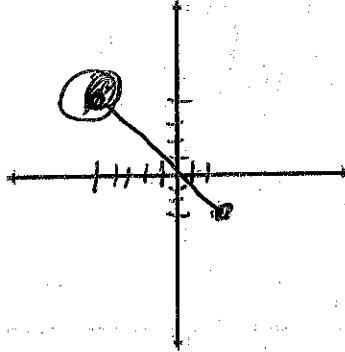
D: _____ R: _____ Function: _____	D: _____ R: _____ Function: _____	D: _____ R: _____ Function: _____
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2. Sketch a graph with the given domain and range:

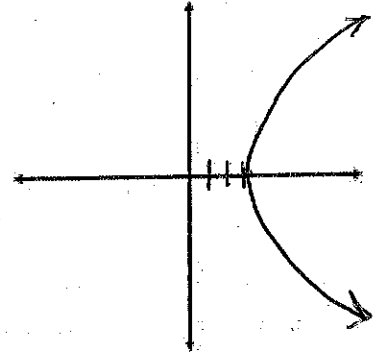
Domain: $3 \leq x \leq 7$
Range: $1 \leq y \leq 10$



Domain: $(-5, 2]$
Range: $[-3, 4)$



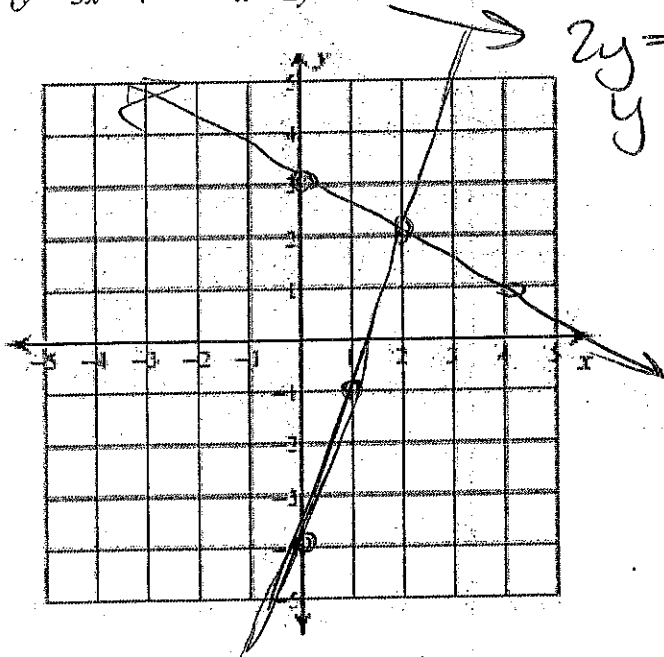
Domain: $[3, \infty)$
Range: $(-\infty, \infty)$



3. Solve the system graphically:

$$\begin{cases} y = 3x - 4 \\ x + 2y = 6 \end{cases}$$

How could you solve this without a graph?



$$\begin{aligned} 2y &= -x + 6 \\ y &= -\frac{1}{2}x + 3 \end{aligned}$$

Substitution

4. Solve the system WITHOUT graphing.

$$\begin{cases} -4x + y = 6 \\ -5x - y = 21 \end{cases}$$

$$\begin{aligned} -4x + y &= 6 \\ + (-5x - y) &= 21 \\ \hline -9x &= 27 \end{aligned}$$

$$x = -3$$

$$\begin{aligned} -4(-3) + y &= 6 \\ 12 + y &= 6 \end{aligned}$$

$$y = -6$$

AA3: Inverses Notes

Questions

Notes

1. I can find the inverse of a function:

A. From an equation:

$$f(x) = \frac{3}{2}x - 1$$

$$g(x) = .5(x+3)^2 - 8$$

$$h(x) = \sqrt{2x+1} + 5$$

$$j(x) = \frac{3}{x-1} + 2$$

Make y =
Solve for x.
Switch x & y

B. From a table.

Switch x & y

x	f(x)	x	f ⁻¹ (x)
-8	-2	-2	-8
-1	-1	-1	-1
0	0	0	0
1	1	1	1
8	2	2	8

C. From a graph

Switch x & y

$$y = \frac{3}{2}x - 1$$

$$y + 1 = \frac{3}{2}x$$

$$\frac{2}{3}(y + 1) = x$$

$$\frac{2}{3}(x + 1) = y^{-1}$$

$$.5(x+3)^2 - 8 = y$$

$$.5(x+3)^2 = y + 8$$

$$\sqrt{x+3} = \sqrt{2y+16}$$

$$x+3 = \sqrt{2y+16}$$

$$x = \sqrt{2y+16} - 3$$

$$y^{-1} = \sqrt{2x+16} - 3$$

$$y = \sqrt{2x+1} + 5$$

$$(y-5) = \sqrt{2x+1}$$

$$(y-5)^2 = 2x+1$$

$$\frac{(y-5)^2 - 1}{2} = x$$

$$\frac{(x-5)^2 - 1}{2} = y^{-1}$$

$$\frac{3}{x-1} + 2 = y$$

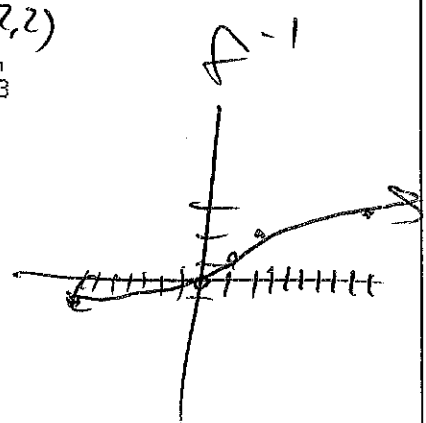
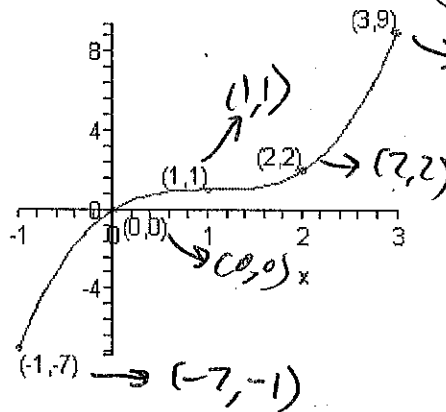
$$\frac{3}{x-1} = y - 2$$

$$3 = (y-2)(x-1)$$

$$\frac{3}{y-2} = x-1$$

$$\frac{3}{y-2} + 1 = x$$

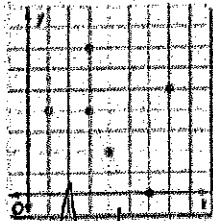
$$\frac{3}{x-2} + 1 = y^{-1}$$



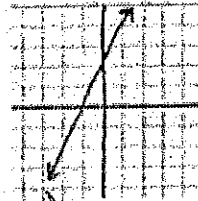
2. I can determine whether or not a relation is a function.

- Using a graph

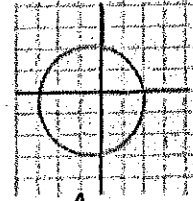
Vertical line Test.



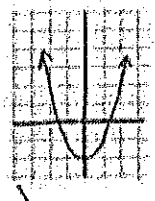
No



Yes



No



Yes

- Using a table

Does each x have only 1 y ?

(No repeat x 's with different y 's)

No

x	y
3	3
4	5
5	7
5	9
6	11

Yes

x	y
5	31
6	28
7	25
8	22
9	19

yes

x	y
2	3
3	3
4	3
5	3
6	3

No

x	y
7	10
8	20
9	30
9	40
10	50

4. I can use composite functions to test inverses (A/B level only)

Use composite function to show that $f(x) = 2\sqrt{x-1} + 2$ and $g(x) = (\frac{x-2}{2})^2 + 1$ are inverses

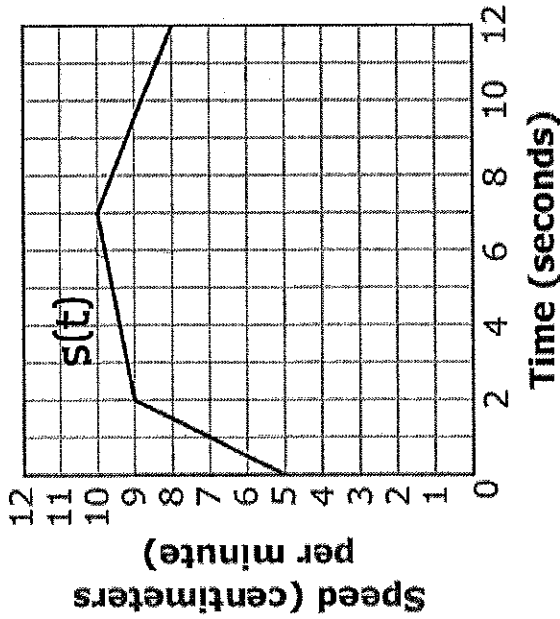
Plug in f to g ,
get x as
result

$$\begin{aligned}
 f(g(x)) &= 2\sqrt{g(x)-1} + 2 \\
 &= 2\sqrt{(\frac{x-2}{2})^2 + 1 - 1} + 2 \\
 &= 2\sqrt{(\frac{x-2}{2})^2} + 2 \\
 &= 2(\frac{x-2}{2}) + 2 \\
 &= x - 2 + 2 \\
 &= x
 \end{aligned}$$

$f \& g$ are inverses

Evaluating a Function From a Graph

Snail Speed as a Function of Time



Evaluate	Meaning
$s(7) = 10$	7 seconds in, speed is 10 cm/min
$s(6) = 9$	6 seconds, 9 cm/min
$s(12) = 10$	After 12 seconds, the snail was traveling at a speed of 10 cm/minute.
$s(11) = 11$	11 seconds, 11 cm/min
$s(1) = 5$	After 1 second(s), the snail was traveling at a speed of 5 cm/minute.

Evaluating a Function From a Table

Charge for Delivery

Miles Traveled	Charge
x	c(x)
1	\$14.00
2	\$21.00
3	\$28.00
4	\$35.00
5	\$42.00
6	\$49.00

Evaluate	Meaning
$c(3) = 28$	3 miles cost \$28
$c(1) = 14$	1 mile costs \$14
$c(5) = 42$	The delivery charge for 5 miles is \$42.00.
$c(4) = 35$	4 miles, \$35
$c(2) = 21$	The delivery charge for 2 miles is \$21.00.

1. Explain the difference between $f(2)$ and $f(x) = 2$.

↑
 $x=2$,
plug in.

↑
 $y=2$, solve
for x .

2. Let $f(x) = 4 - 2x$

a) Evaluate $f(-6)$

$$4 - 2(-6)$$

$$4 + 12$$

$$16$$

b) Evaluate $f(3a)$

$$4 - 2(3a)$$

$$4 - 6a$$

c) Evaluate $f(t+2)$

$$4 - 2(t+2)$$

$$4 - 2t - 2$$

$$2 - 2t$$

d) Solve $f(x) = 5$

$$4 - 2x = 5$$

$$-4 \quad -4$$

$$-2x = 1$$

$$\frac{-2x}{-2} = \frac{1}{-2}$$

$$x = -\frac{1}{2}$$

3. Let $g(x) = x^2 - 7$

a) Evaluate $g(-3) = (-3)^2 - 7$

b) Solve $g(x) = -6$ $9 - 7 = 2$

$$b) x^2 - 7 = -6$$

$$+7 \quad +7$$

$$x^2 = 1$$

$$x = \pm 1$$

4. Let $h(x) = (x-2)(x+7)$

a) Evaluate $h(2)$ $h(2) = (2-2)(2+7)$

b) Evaluate $h(a)$ $= (0)(9)$

$$h) h(a) = (a-2)(a+7)$$

5. Let $f(x) = \frac{8}{x+2}$

a) Evaluate $f(14) = \frac{8}{14+2} = \frac{1}{2}$

b) Evaluate $f(t) = \frac{8}{t+2}$

c) Solve $f(x) = 1$

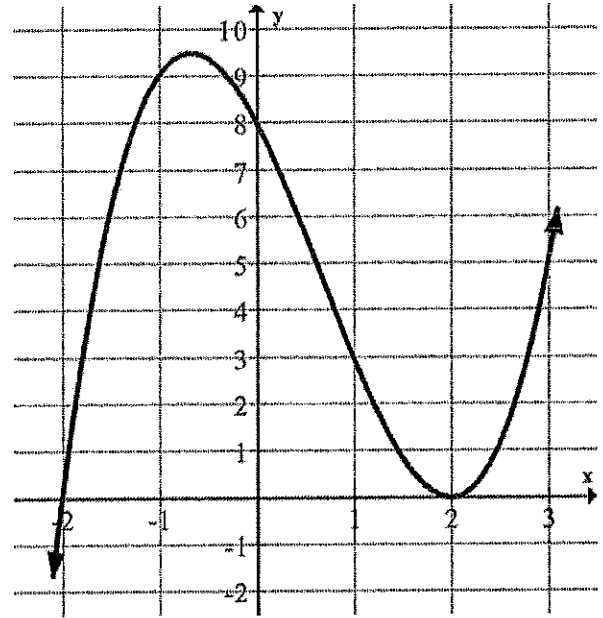
$$\left(\frac{8}{x+2} = 1\right) \times +2$$

$$8 = x+2$$

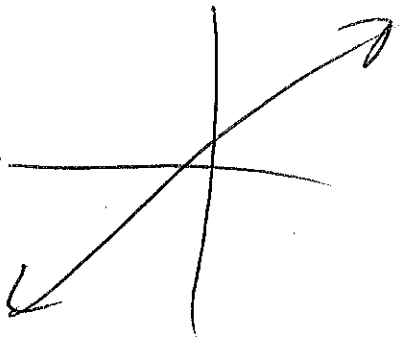
$$6 = x$$

6. Use the graph of $f(x)$ below to answer the following questions.

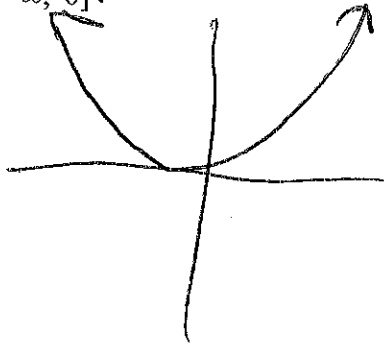
- a) Evaluate $f(3)$ 5
- b) Evaluate $f(-1)$ 9
- c) Solve $f(x) = 0$ $x = -2, 2$
- d) Solve $f(x) = -1$ $x = -2.1$
- e) Identify the domain of this function.
- f) On what interval is the function decreasing? $(-0.5, 2)$
- g) On what interval is the function increasing? $(-\infty, -0.5)$ $(2, \infty)$
- h) Does the function have an absolute maximum? No



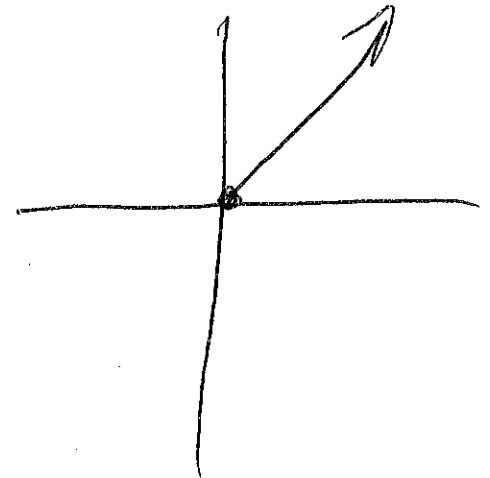
8. Sketch the graph of a function whose domain is $(-\infty, \infty)$ and whose range is $(-\infty, \infty)$.



9. Sketch the graph of a function whose domain is $(-\infty, \infty)$ and whose range is $(-\infty, 0]$.



10. Sketch the graph of a function whose domain and range are both $[0, \infty)$.



11. Use the table of values to answer the questions below.

x	-7	-2	0	1	3	4	6
$f(x)$	6	3	0	-2	1	0	0

a. Evaluate $f(3) = 1$

c. Solve $f(x) = 6$
 $x = -7$

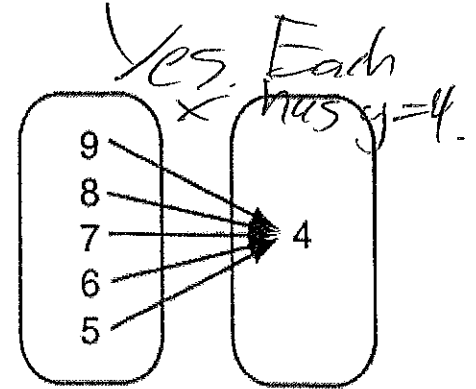
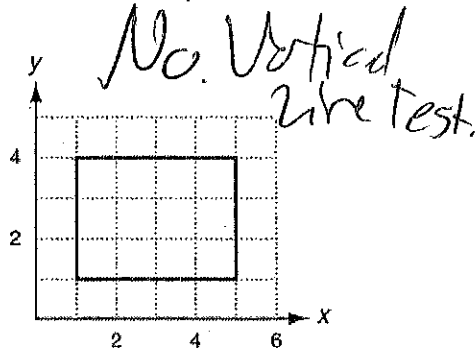
b. Evaluate $f(6) = 0$

d. Solve $f(x) = 0$
 $x = 0, 4, 6$

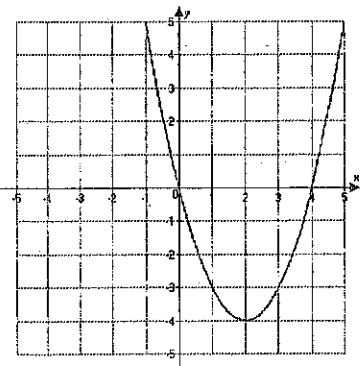
1. Tell whether the following are functions. Explain.

$\{(-2, 5), (-1, 1), (3, 1), (-1, -2)\}$

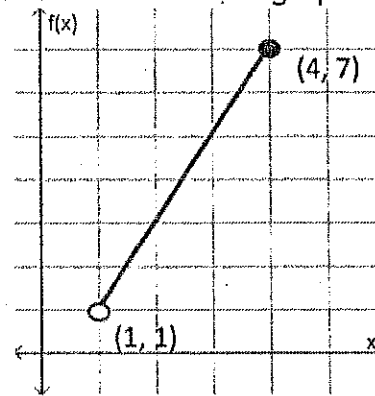
No. $x = -1$ has
2 y 's



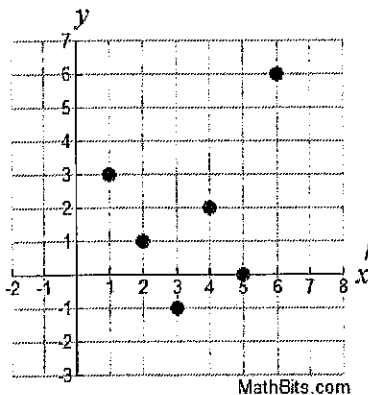
2. Find domain and range of the given graphs below. State if each graph is a function:



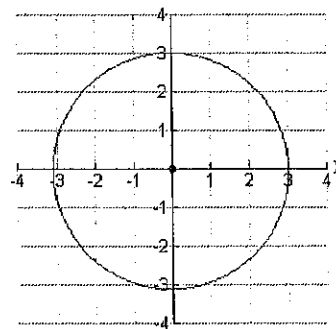
D: $(-\infty, \infty)$
R: $[-4, \infty)$
F: Yes



D: $(1, 4]$
R: $(1, 7]$
F: Yes



D: $\{1, 2, 3, 4, 5, 6\}$
R: $\{-1, 0, 1, 2, 3, 6\}$
F: Yes



D: $[-3, 3]$
R: $[-3, 3]$
F: No

3. Use the functions $f(x) = x^2 - 4$, $g(x) = x - 3$, $h(x) = x + 2$ to answer the questions below.

a. Evaluate $f(-7)$

b. Solve $h(x) = -7$

c. Evaluate $g(-4)$

d. Solve $g(x) = 1$

e. Solve $f(x) = -5$

f. Evaluate $h(-1)$

g. Find the domain of $f(x)$.

h. Find the range of $h(x)$

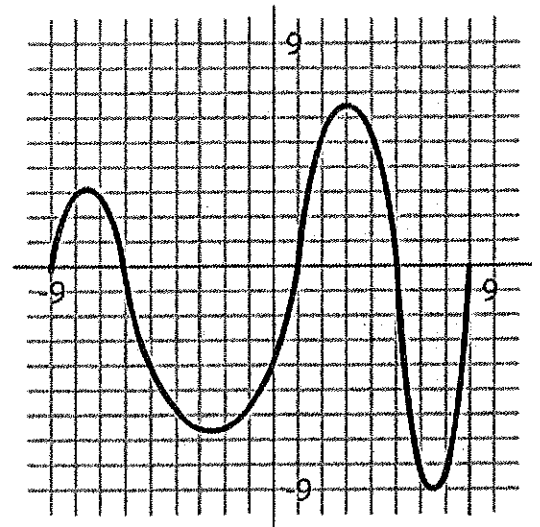
i. Find the range of $g(x)$

4. The following graph completely defines $f(x)$.

a. Evaluate $f(2)$

b. Evaluate $f(-2)$

c. Solve $f(x) = 0$



5. Find the inverse of the following functions:

a. $y = \frac{1}{2}x - 3$
 $+3 \quad +3$

$$2(y+3) = \frac{1}{2}x \cdot 2$$

$$2y+6 = x$$

$$2y+6 = y^{-1}$$

b. $g(x) = \sqrt[3]{x} + 3$

$$y = \sqrt[3]{x} + 3$$

$$(y-3)^3 = (\sqrt[3]{x})^3$$

$$(y-3)^3 = x$$

$$(x-3)^3 = y^{-1}$$

c. $h(x) = \frac{7x+18}{2}$

$$y = \frac{7x+18}{2}$$

$$2y = 7x+18$$

$$2y-18 = 7x$$

$$\frac{2y-18}{7} = x$$

$$\frac{2x-18}{7} = y^{-1}$$

d. $f(x) = 2x^4 + 5$

$$y = 2x^4 + 5$$

$$y-5 = 2x^4$$

$$\sqrt[4]{\frac{y-5}{2}} = \sqrt[4]{x^4}$$

$$\sqrt[4]{\frac{y-5}{2}} = x$$

$$\sqrt[4]{\frac{x-5}{2}} = y^{-1}$$

6. Given two function machines $f(x) = x^2 - 1$ and $g(x) = 3(x+2)$.

a. If the two machines are connected so that $f(x)$ comes first, and 5 is dropped in, what comes out? (This is finding $g(f(5))$)

$$f(5) = 5^2 - 1 = 24$$

$$g(24) = 3(24+2) = 3(26) = 78$$

a. If the two machines are connected so that $g(x)$ comes first, and 5 is dropped in, what comes out? (This is finding $f(g(5))$)

$$g(5) = 3(5+2) = 3(7) = 21$$

$$f(21) = 21^2 - 1 = 441 - 1 = 440$$

7. Given two function machines $f(x) = \frac{2}{x-7}$ and $g(x) = 2x + 5$ calculate:

b. $g(3) = 2(3) + 5$
 $6 + 5 = 11$

d. $f(g(2)) = f(2(2) + 5)$
 $f(4 + 5)$
 $f(9)$
 $\frac{2}{9-7} = \frac{2}{2} = 1$

c. $f(10) = \frac{2}{10-7} = \frac{2}{3}$

e. $g(f(11)) = g(\frac{2}{11-7})$
 $g(\frac{2}{4})$
 $2(\frac{2}{4}) + 5$
 6

8. Find and verify the inverse of the following functions:

a. $f(x) = 3(4x + 5) - 1$

$$y + 1 = 3(4x + 5)$$

$$\frac{y+1}{3} = 4x + 5$$

$$\frac{y+1}{3} - 5 = 4x$$

$$\frac{y+1}{3} - 5 = x$$

$$\frac{\frac{y+1}{3} - 5}{4} = y^{-1}$$

Verify: $f(0) = 3(4(0) + 5) - 1$
 $= 3(5) - 1$
 $= 14$

$$f^{-1}(14) = \frac{14+1}{3} - 5$$

$$= \frac{15}{3} - 5$$

$$= \frac{5}{1} - 5$$

$$= 5 - 5 = 0 \quad \checkmark$$

b. $g(x) = \frac{\sqrt[3]{x+4}}{2}$

$$2y = \sqrt[3]{x+4}$$

$$(2y)^3 = x + 4$$

$$(2y)^3 - 4 = x$$

$$(2x)^3 - 4 = y^{-1}$$

$$g(0) = \frac{\sqrt[3]{0+4}}{2} = \frac{\sqrt[3]{4}}{2}$$

$$g^{-1}\left(\frac{\sqrt[3]{4}}{2}\right) = (2\left(\frac{\sqrt[3]{4}}{2}\right))^3 - 4$$

$$= (\sqrt[3]{4})^3 - 4$$

$$= 4 - 4$$

$$= 0 \quad \checkmark$$

1. Use the method from the previous class to find the inverse of each function below. Show your work.

a. $L(x) = 5x - 1$

$$y + 1 = 5x$$

$$\frac{y+1}{5} = x$$

$$5$$

$$\frac{x+1}{5} = y^{-1}$$

b. $s(x) = \sqrt{x+4} - 3$

$$y = \sqrt{x+4} - 3$$

$$y + 3 = \sqrt{x+4}$$

$$(y+3)^2 = x+4$$

$$(y+3)^2 - 4 = x$$

$$(x+3)^2 - 4 = y^{-1}$$

2. a. Complete the table below for the relation $g(x) = (x-4)^2 + 1$

x	-2	0	2	4	6
y	37	17	5	1	5

b. Based on the table, is $g(x)$ a function? Explain why or why not.

Yes

c. Complete the table below for the **inverse** of $g(x)$.

x	37	17	5	1	5
y	-2	0	2	4	6

d. Is the inverse of $g(x)$ a function? Explain why or why not.

No. $x=5$ has $y=2$ & $y=6$.

1. Use the method from the previous class to find the inverse of each function below. Show your work.

a. $L(x) = 3x - 7$

$$y = 3x - 7$$

$$y + 7 = 3x$$

$$\frac{y + 7}{3} = x$$

$$L^{-1}(x) = \frac{x + 7}{3}$$

b. $s(x) = \sqrt{x+7} - 2$

$$s(x) - 2 = \sqrt{x+7}$$

$$(y-2)^2 = x+7$$

$$(y-2)^2 - 7 = x$$

$$(x-2)^2 - 7 = y^{-1}$$

2. a. Complete the table below for the relation $g(x) = (x-3)^2 + 4$

x	-1	1	3	5	7
y	20	8	4	8	20

b. Based on the table, is $g(x)$ a function? Explain why or why not.

Yes Each x has 1 y .

c. Complete the table below for the **inverse** of $g(x)$.

x	20	8	4	8	20
y	-1	1	3	5	7

d. Is the inverse of $g(x)$ a function? Explain why or why not.

No. $x=8$ has $y=1$ & $y=5$

Function Inverses

State if the given functions are inverses.

1) $g(x) = 4 - \frac{3}{2}x$

$f(x) = \frac{1}{2}x + \frac{3}{2}$

No $g(0) = 4$
 $f(4) \neq 0$

2) $g(n) = \frac{-12 - 2n}{3}$

$f(n) = \frac{-5 + 6n}{5}$

No
 $g(0) = -4$
 $f(-4) \neq 0$

3) $f(n) = \frac{-16 + n}{4}$

$g(n) = 4n + 16$

$f(0) = -4$
 $g(-4) = 0$
Yes

4) $f(x) = -\frac{4}{7}x - \frac{16}{7}$

$g(x) = \frac{3}{2}x - \frac{3}{2}$

No
 $f(0) = -\frac{16}{7}$
 $g(-\frac{16}{7}) \neq 0$

5) $f(n) = -(n+1)^3$

$g(n) = 3 + n^3$

$f(0) = -(0+1)^3 = -1$
 $g(-1) = 3 + (-1)^3 \neq 0$
No

6) $f(n) = 2(n-2)^3$

$g(n) = \frac{4 + \sqrt[3]{4n}}{2}$

Yes
 $f(0) = 2(0-2)^3 = -16$
 $g(-16) = \frac{4 + \sqrt[3]{-64}}{2} = 0$

7) $f(x) = \frac{4}{-x-2} + 2$

$h(x) = -\frac{1}{x+3}$

No
 $f(0) = 0$
 $h(0) \neq 0$

8) $g(x) = -\frac{2}{x} - 1$

$f(x) = -\frac{2}{x+1}$

No
 $g(1) = -3$
 $f(-3) \neq 1$

Find the inverse of each function.

9) $h(x) = \sqrt[3]{x} - 3$

$h^{-1}(x) = (x+3)^3$

10) $g(x) = \frac{1}{x} - 2$

$g^{-1}(x) = \frac{1}{x+2}$

11) $h(x) = 2x^3 + 3$

$h^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$

12) $g(x) = -4x + 1$

$g^{-1}(x) = \frac{x-1}{-4}$

13) $g(x) = \frac{7x+18}{2}$

$g^{-1}(x) = \frac{2x-18}{7}$

14) $f(x) = x+3$

$f^{-1}(x) = x-3$

15) $f(x) = -x+3$

$f^{-1}(x) = -x+3$

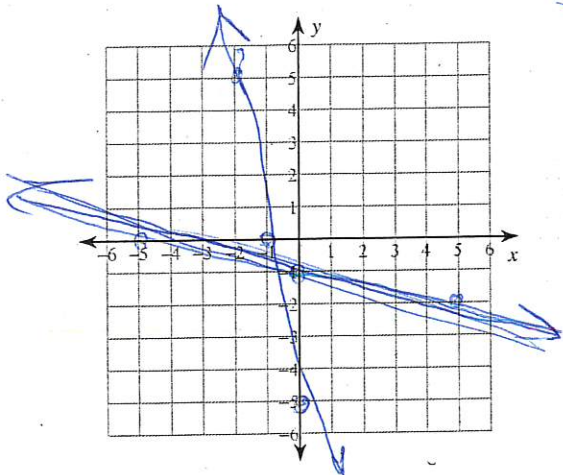
16) $f(x) = 4x$

$f^{-1}(x) = \frac{1}{4}x$

Find the inverse of each function. Then graph the function and its inverse.

17) $f(x) = -1 - \frac{1}{5}x$

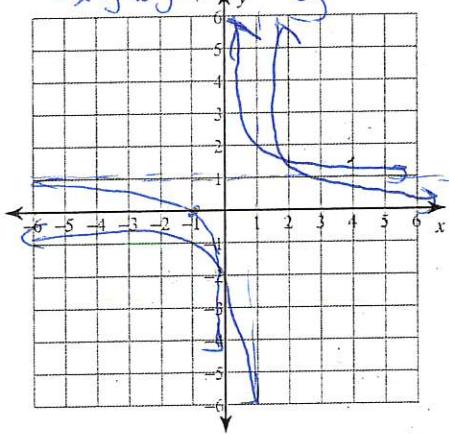
$f^{-1}(x) = -5(x+1)$
 $-5x-5$



18) $g(x) = \frac{1}{x-1}$

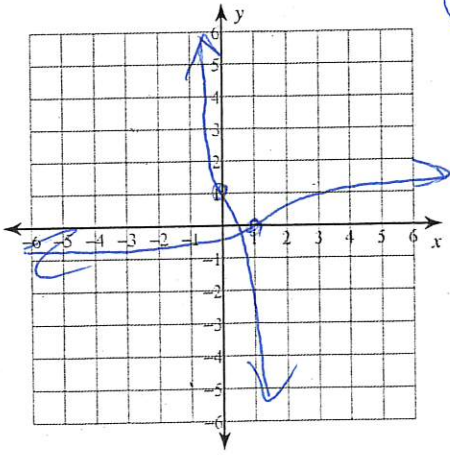
$g^{-1}(x) = -\frac{1+x}{x}$

$y(x-1) = 1$
 $xy - y = 1$
 $xy = y+1$
 $x = \frac{y+1}{y}$



19) $f(x) = -2x^3 + 1$

$f^{-1}(x) = \sqrt[3]{\frac{x-1}{-2}}$



20) $g(x) = \frac{-x-5}{3}$

$g^{-1}(x) = -3x+5$
 $-3x-5$

