

1. Greta is hatching a plan to buy a house and move to Bora Bora. She knows (based on her newly acquired knowledge of compound interest) that if she invests now, her money will grow over time using the equation $V(x) = V(0)(1+r)^x$, where $V(x)$ = value of her money after x years, $V(0)$ = initial amount invested and r = the interest rate she will earn.

- a. Explain why the calculation uses $1 + r$ to calculate the growth in her account

- b. Greta does research and finds that one of the highest yielding stocks for 2016 (Fifth Street Financial) paid 13% annual return. If she invested \$1000 now at that rate, what would be the value of her investment after 20 years?

- c. Okay, so it turns out that Bora Bora real estate is expensive:
<http://www.sothebysrealty.com/eng/sales/pyf> . Greta figures that she should be able to get something decent for \$500,000. Determine how much she would need to invest now (still at 13% for 20 years) to afford a \$500,000 property.

- d. That seems like an unreasonable amount. How about this: What interest rate would she need to earn to turn her \$1000 into enough money to afford a \$500,000 property after 20 years?

- e. Hmm...I don't think there has ever been an investment that pays that percent of return. How many years would Greta need to keep her investment of \$1000 at 13% to afford a \$500,000 property? (Why can't you solve this equation like the previous equations?)

2. Written mathematics is a language with specific vocabulary and grammar. Symbols have been invented to mean particular things just like words and punctuation. Many of the symbols are so familiar to people that they don't have to think about them. Use what you know about the mathematical symbols used below to solve each equation:

a. $x + 5 = 6$

b. $3x = 12$

c. $\frac{x}{2} = 10$

d. $x^3 = 8$

e. $3^x = 9$

f. $2^x = 16$

3. Sometimes, the language of mathematics requires additional inventions (symbols) to solve equations. A good example of this is exponents/roots. Roots were defined as the inverse of raising different values to a given power, as follows:

$$y = x^n \text{ is equivalent to } x = \sqrt[n]{y}$$

In other words, people just made up the word "root" and the symbol " $\sqrt[n]{x}$ " because they wanted an efficient way of writing "The number that is the inverse of the nth power"

Convert each equation below to the root version. Then use a calculator to solve for x:

a. $x^4 = 5.0625$

b. $x^{2.5} = 1024$

c. $x^{\frac{1}{4}} = 81$

Convert each equation below to the exponent version. Then use a calculator to solve for x:

d. $\sqrt[2]{x} = 25$

e. $\sqrt[12]{x} = 1$

d. $\sqrt[4]{x} = 32$

4. Referring back to question 1(e),

a. explain why the equation you need to solve is

$$500000 = 1000(1.13)^x$$

b. The issue with solving this equation is that you haven't yet learned the language for reversing an equation when the exponent is a variable... until now!

Logarithms¹ are defined as the inverse of raising a given number to various exponents as follows:

$$y = a^x \text{ is equivalent to } x = \log_a y, \text{ where } a \text{ is called the base of the logarithm.}$$

To calculate your answer, use the change of base formula: $\log_a y = \frac{\log(y)}{\log(a)}$

Convert each equation below to the logarithm version and use a calculator to solve for x.

i. $3^x = 243$

ii. $10^x = 0.01$

iii. $2.4^x = 33.1776$

Convert each equation to the exponential version and determine the value of x.

iv. $\log_3 x = 2$

v. $\log_5 25 = x$

vi. $\log_2 x = 0$

c. Return to question 1(e) and solve the equation using logarithms.

¹ Logarithms weren't invented until 1614 and, before the use of calculators, relied on tables constructed by meticulous calculations using base 10 logarithms. Today, scientific calculators will calculate logarithms of any base.

Recall the Compound Interest Formula: $F(x) = F(0)(1 + r)^x$

1. Understanding the formula

- a. Why does $100(1.03)^t$ mean you are earning 3% interest?
- b. What does $F(0)$ represent about your bank account?
- c. What interest rate do you earn if your equation is $f(x) = 350(1.04)^x$?
- d. What interest rate do you earn if your equation is $f(x) = 1350(1.025)^x$?
- e. What interest rate do you earn if your equation is $f(x) = 35000(2)^x$?

2. Applying the formula

- a. Find the amount of money in your account if you invest \$1000 at 5% interest for 10 years.
- b. Find the amount of money in your account if you invest \$2500 at 8% interest for 5 years.
- c. Find the amount of money in your account if you invest \$15000 at 2.5% interest for 12.5 years.

3. Solving with the formula

- a. How much money do you need to invest to have \$500,000 in 10 years at 4% interest?
- b. What interest rate do you need to earn to have \$500,000 in 10 years if you start with \$10,000?
- c. How long will it take \$10,000 to grow to \$500,000 if you earn 4% interest?

4. Use what you've learned to practice the following questions:

a. A bank account pays interest on a savings account according to the equation

$$F = P(1.02)^t, \text{ if } 0 < P < 1000$$

$$= P(1.04)^t, \text{ if } 1000 \leq P < 5000$$

$$= P(1.06)^t, \text{ if } 5000 \leq P$$

Explain in words the bank's interest payment plan.

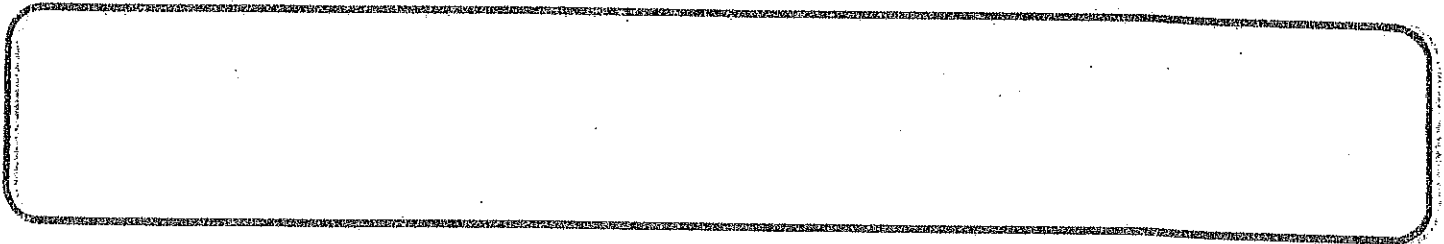
b. Describe in words, the exponential pattern represented by the equation $y = 40 \cdot 0.8^x$

c. Write an equation to fit the table shown.

| x_1 | y_1 |
|-------|--------|
| 0 | 100 |
| 5 | 70 |
| 10 | 49 |
| 15 | 34.3 |
| 20 | 24.01 |
| 25 | 16.807 |

d. Write an equation to fit the situation: Bennett puts \$5,000 in a Certificate of Deposit that pays 3.25% interest every 2 years.

e. How much money would Bennett have in his Certificate of Deposit after 20 years?



Rewrite each equation in exponential form:

1. $\log_{15} 225 = 2$

3. $\log_3 243 = 5$

2. $\log_{11} 121 = 2$

4. $\log_{216} 6 = \frac{1}{3}$

Rewrite each equation in logarithmic form:

5. $3^2 = 9$

7. $8^3 = 512$

6. $49^{\frac{1}{2}} = 7$

8. $10^5 = 10000$

Rewrite in exponential form and solve for x, y, or b, if possible:

9. $\log_2 32 = y$

12. $\log_2 x = -5$

10. $\log_2 \frac{1}{2} = y$

13. $\log_b 49 = 2$

11. $\log_2 x = 0$

14. $\log_b \frac{1}{27} = -3$

Evaluate each expression:

15. $\log_4 64$

16. $\log_6 216$

17. $\log_4 16$

18. $\log_3 \frac{1}{243}$

19. $\log_5 125$

Solve each equation:

25. $19^{10x} = 84$

26. $6^{x+8} = 32$

27. $2(3^x) = 120$

20. $\log_2 4$

21. $\log_{343} 7$

22. $\log_2 16$

23. $\log_{64} 4$

24. $\log_6 \frac{1}{216}$

28. $10(2^x) + 15 = 315$

29. $10 \log(6x) = -10$

30. $-4 \log(-7x) = -12$

Name _____

C Level Questions

1. Solve each equation for x (round your answers to nearest hundredth). Show all of your work.

a. $2^x = 100$

b. $\log_5(x - 1) = 2.1$

c. $8\log_2(4x) - 5 = 75$

d. $5 \cdot 4^{x-3} + 10 = 10$

2. Find the inverse of

a. $y = 2\log_8\left(\frac{x}{3}\right) - 5$

b. $y = 5 \cdot 4^{x-2} + 3$

3. Ami opened an account at Advantis Credit Union. The balance of her account after x years is represented by the function $b(x) = 500(1.05)^x$.

a. What interest rate is Ami earning?

b. After how many years did it take for Ami's balance to double?

C Level Questions

1. Solve each equation for x (round your answers to nearest hundredth). Show all of your work.

a. $3^x = 100$

b. $\log_5(x - 3) = 1.4$

c. $8\log_2(4x) - 9 = 31$

d. $5 \cdot 4^{x-3} + 10 = 15$

2. Find the inverse of

a. $y = 3\log_5\left(\frac{x}{2}\right) - 4$

b. $y = 3 \cdot 6^{x-1} + 2$

3. Ami opened an account at Advantis Credit Union. The balance of her account after x years is represented by the function $b(x) = 400(1.04)^x$.

a. What interest rate is Ami earning?

b. After how many years did it take for Ami's balance to double?

1. A Super Ball rebounds to $\frac{3}{4}$ of its previous height after each bounce. If you drop a Super Ball from a height of 20 m, after how many bounces will it reach a height of 0.0098 m? Solve using an exponential model and logarithms.

2. Reproduction of an African Dung Beetle is the focus of a laboratory experiment. There were 25 Dung Beetles at the beginning of the experiment. It was noted that the number of Dung Beetles increase 3% every 28 days. After how many days will there be 500 Dung Beetles? Solve using an exponential model and logarithms.

3. Two rival companies: Acme Lighting and Bargain Bulbs decided to make the same LED light bulbs using two different processes. The revenue (in thousands of \$) of the two companies are represented by: $a(t) = 1000 \log_4 t + 100$ and $b(t) = 1200 \log_5 t$ where t = time in months.
 - i. How many months will it take for Acme Lighting to have \$5,000,000 in revenue? Show how you found your answer.

 - ii. How many months will it take Bargain Bulbs to have \$5,000,000 in revenue? Show how you found your answer.

 - iii. Using the graphs of $a(t)$ and $b(t)$, after how many months will Acme Lighting and Bargain Bulbs have the same revenue?

 - iv. Find the inverse of $a(t)$.

 - v. Evaluate $a^{-1}(5,000,000)$. What does this mean about Acme Lighting's revenue?

4. Radium (Ra) is a radioactive element that decays as follows:
 In 3,000 years, a 100 gram sample of radium decays to a mass of 27.04 grams.
 - a. Write an exponential function to describe the decay of radium over time. Define your variables.

 - b. Find the inverse of the exponential function from part (a).

 - c. Use the inverse function to determine the number of years it would take a sample of radium to decay to half of its original mass.

5. The towns of Geometrix and Matrix are matched for a cultural exchange. The population of Geometrix is 40,000 while the population of Matrix is 10,000. For the next 30 years, experts predict that the population of Geometrix will decline by 3% per year. During the same period, they expect that the population of Matrix will increase by 5% annually.

a. Find after how many years the two towns will have the same population graphically.

b. How many years ago did Geometrix have a population of 10,000? Show how you found your answer.

6. When interest is paid n times a year, the value of an initial investment, P , that collects an annual interest rate of r (as a decimal) for x years can be represented by the function

$$C(x) = P\left(1 + \frac{r}{n}\right)^{nx}$$

Ella wants to invest \$2000. She has two investment options:

| | |
|--|--|
| <p>Investment Option A</p> <ul style="list-style-type: none"> • Annual Interest Rate of 5% • Interest Paid Once per Year | <p>Investment Option B</p> <ul style="list-style-type: none"> • Annual Interest Rate of 4.2% • Interest Paid Monthly (12 times per year) |
|--|--|

Using the graphs of the functions representing each Investment Option, determine after how many months, the two Investments Options will have the same balance?

7. An airplane is flying at an altitude of 10,000 meters. At 21:00, the pilot begins the descent towards PDX Airport. The descent follows an exponential model, $d(x)$, ending with the plane's landing. At 21:04, the airplane is an an altitude of 5,222 meters.

a. At what time will the plane be at 280 meters? Solve using logarithms.

b. Find the inverse of the exponential function that represents the airplane's descent, $d^{-1}(x)$.

c. Evaluate $d^{-1}(280)$. What does this tell you about the airplane's descent?

*****COMPLETE ON A SEPARATE SHEET OF PAPER*****

- Aaron invests \$20,000 at 5% interest. How much does he have in the account after 15 years?
- The value of a new \$20,000 minivan depreciates 15% per year. Find its value after 5 years.
- For the function machine $f(x) = 3^x$:
 - Find $f(2)$
 - Find $f(-1)$
 - If 81 came out, what went in?
 - If 0 came out, what went in?
 - If 8 came out, what went in?
- For each equation below, solve for x. Show all steps!
 - $5x^3 = 80$
 - _____ = _____
 - _____ = _____
- Rewrite in exponential form and solve.
EXAMPLE:
 $\log_2 64 = y$ ----->> $2^y = 64$, or "What power of 2 is the number 64?". Answer: 6
 - $\log_2 8 = y$
 - $\log_4 64 = y$
 - $\log_2 x = -6$
 - $\log_3 \frac{1}{3} = y$
 - $\log_2 x = -5$
 - $\log_x \frac{1}{2} = 8$
- Rewrite in logarithmic form and solve.
 - $20 = 2(7^x) - 6$
 - $15.3 = 5^{(x+2)}$
 - $75 - 5(3^x) = -150$
- Do you need to change the form of this equation to solve? Show how to solve.
 - $\log_4 64 = x - 1$
 - $\log_4 \frac{1}{64} = x$
 - $\log_9(-11x + 2) = \log_9(x^2 + 30)$
 - $\log(5x) = \log(2x + 9)$

8. Mixed Review. Solve.

a. $-6 \log_3(x - 3) = -24$

b. $\log_5 \frac{1}{125} = x$

c. $\log_y \frac{1}{27} = -3$

d. $8^x = 190$

9. Can the value of $\log_2(-4)$ be found? What about the value of $\log_2 0$? Why or why not? What does this tell you about the domain of $\log_b x$?

10. You get \$500 for your 18th birthday and decide to open a savings account. You find an amazing bank that will guarantee an interest rate of 6.5%.

a. Write an exponential equation to model the situation.

b. How much money will be in the account when you are 30? Solve algebraically.

c. How old will you be when your account is worth \$10,000? Solve algebraically.

11. The number of fish in a pond is 150. The fish population is growing exponentially at a rate of 15.5% a month.

a. Write an exponential equation to model the situation.

b. How many fish will be in the pond after a year?

c. How long will it take for the population to reach 10,000 fish? Solve algebraically.

12. The mosquitoes in the campground are decreasing exponentially. After 10 minutes there are 880 mosquitoes. After 14 minutes there are only 460 mosquitoes.

a. Write an exponential equation to model the situation.

b. Assuming the mosquitoes continue to decrease at this rate, how long will it take until there are only 50 mosquitoes left? Solve algebraically.