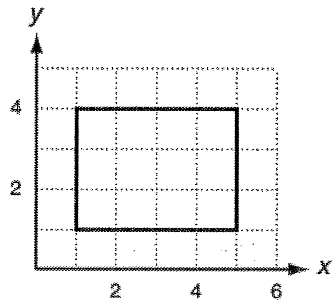
Algebra 3-4
Unit 3 Review: Functions & Inverses

Name: Key Per:_____
Date: 9-20

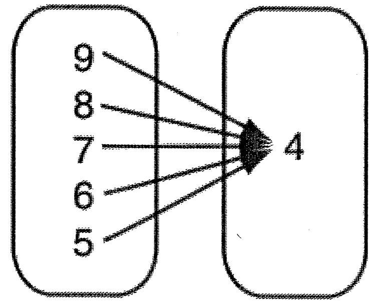
1. Tell whether the following are functions. Explain.

$\{(-2,5), \underline{(-1,1)}, (3,1), \underline{(-1,-2)}\}$

no, not a function
The x-value -1 has two y-values (1 and -2)

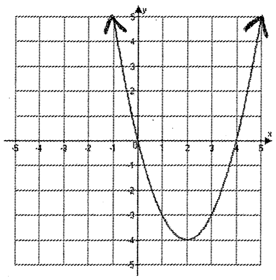


no, not a function
all x-values have more than one y-value

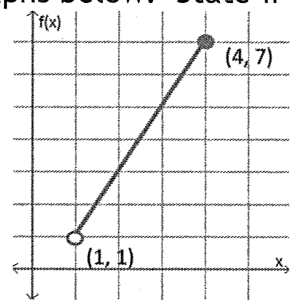


yes, is a function.
No x-value has more than one y-value.

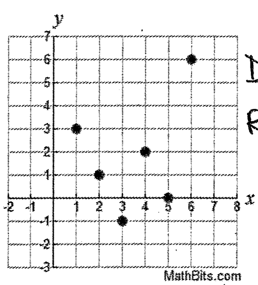
2. Find domain and range of the given graphs below. State if each graph is a function:



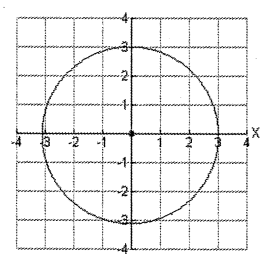
D: $-\infty < x < \infty$ $(-\infty, \infty)$
R: $-4 \le y < \infty$ $[-4, \infty)$
yes, a function



D: $1 < x \le 4$ $(1, 4]$
R: $1 < y \le 7$ $(1, 7]$
yes, is a function

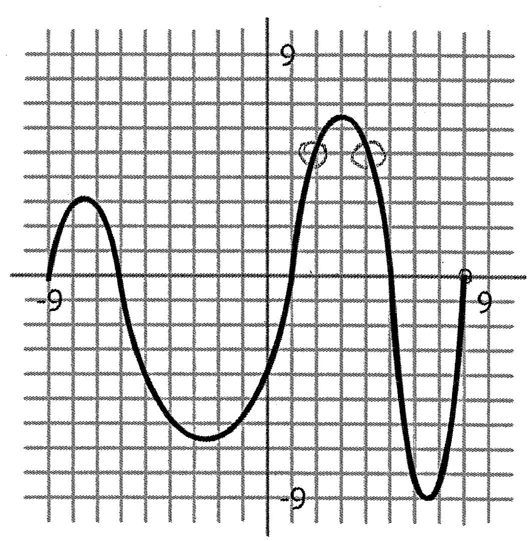


D: $\{1, 2, 3, 4, 5, 6\}$
R: $\{-1, 0, 1, 2, 3, 6\}$
yes, is a function



D: $-3 \le x \le 3$ $[-3, 3]$
R: $-3 \le y \le 3$ $[-3, 3]$
no, not a function

3. **The following graph completely defines f(x).**



a. Evaluate $f(8)$

(when x is 8, what is the y value?)
$f(8) = 0$

b. Evaluate $f(0) - f(8)$

$f(0) = -4$
$f(8) = 0$

$-4 - 0 = -4$

c. Solve $f(x) = 5$

(when y is 5, what is the x-value?)

$x = 2$ and $x = 4$

4. Use the functions $f(x) = 2|x| - 5$, $g(x) = x^2 - 3$, $h(x) = 3x + 5$ to answer the questions below.

a. Evaluate *f(-7)*

$f(-7) = 2|-7| - 5$

$f(-7) = 2(7) - 5$

$f(-7) = 14 - 5$

$\boxed{f(-7) = 9}$

b. Solve *h(x)* = -7

$-7 = 3x + 5$

$-12 = 3x$

$\boxed{-4 = x}$

c. Evaluate *g(-4)*

$g(-4) = (-4)^2 - 3$

$g(-4) = 16 - 3$

$\boxed{g(-4) = 13}$

d. Solve *g(x)* = 1

$1 = x^2 - 3$

$4 = x^2$

$\boxed{\pm 2 = x}$

e. Solve *f(x)* = -5

$-5 = 2|x| - 5$

$0 = 2|x|$

$0 = |x|$

$\boxed{0 = x}$

f. Evaluate *h(-1)*

$h(-1) = 3(-1) + 5$

$h(-1) = -3 + 5$

$\boxed{h(-1) = 2}$

g. Find the domain of *f(x)*.

$D: -\infty < x < \infty$

$(-\infty, \infty)$

h. Find the range of *h(x)*

$R: -\infty < y < \infty$

$(-\infty, \infty)$

i. Find the range of *g(x)*

$R: -3 \le y < \infty$

$[-3, \infty)$

5. Given two function machines $f(x) = x^2 - 1$ and $g(x) = 3(x + 2)$.

a. If the two machines are connected so that $f(x)$ comes first, and 5 is dropped in, what comes out? (This is finding $g(f(5))$)

$5 \rightarrow \boxed{f(x)} \rightarrow \boxed{g(x)} \rightarrow ?$

$g(f(5)) = g(24) = 3(24+2) = 78$

$f(5) = (5)^2 - 1 = 24$

a. If the two machines are connected so that $g(x)$ comes first, and 5 is dropped in, what comes out? (This is finding $f(g(5))$)

$5 \rightarrow \boxed{g(x)} \rightarrow \boxed{f(x)} \rightarrow$

$f(g(5)) = f(21) = (21)^2 - 1 = 440$

$g(5) = 3(5+2) = 21$

6. Find the inverse of the following functions:

a. $y = \frac{1}{2}x - 3$

$y+3 = \frac{1}{2}x$

$2(y+3) = x$

$\boxed{2(x+3) = y^{-1}}$

c. $h(x) = \frac{7x+18}{2}$

$2h(x) = 7x+18$

$2h(x) - 18 = 7x$

$\frac{2h(x)-18}{7} = x$

$\boxed{\frac{2x-18}{7} = h^{-1}(x)}$

b. $g(x) = \sqrt[3]{x} + 3$

$g(x) - 3 = \sqrt[3]{x}$

$(g(x)-3)^3 = x$

$\boxed{(x-3)^3 = g^{-1}(x)}$

d. $f(x) = 2x^4 + 5$

$f(x) - 5 = 2x^4$

$\frac{f(x)-5}{2} = x^4$

$\pm\sqrt[4]{\frac{f(x)-5}{2}} = x$

$\boxed{\pm\sqrt[4]{\frac{x-5}{2}} = f^{-1}(x)}$

7. Given two function $f(x) = \frac{2}{x-7}$ and $g(x) = 2x + 5$ calculate:

a. $g(3) =$

$g(3) = \frac{2}{3-7}$

$g(3) = \frac{2}{-4}$

$\boxed{g(3) = -\frac{1}{2}}$

c. $f(g(2)) = f(9) = \frac{2}{9-7} = \frac{2}{2} = \boxed{1}$

$g(2) = 2(2) + 5 = 9$

b. $f(10) = \frac{2}{10-7}$

$\boxed{f(10) = \frac{2}{3}}$

d. $g(f(11)) = g\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 5 = \boxed{6}$

$f(11) = \frac{2}{11-7} = \frac{2}{4} = \frac{1}{2}$

8. Find and verify the inverse of the following functions:

a. $f(x) = 3(4x+5) - 1$ $= 12x + 15 - 1 = 12x + 14$

$f(x) - 14 = 12x$

$\frac{f(x)-14}{12} = x$

$\frac{x-14}{12} = f^{-1}(x)$

$f^{-1}(f(x)) = \frac{12x+14-14}{12}$

$= \frac{12x}{12}$

$= x$

b. $g(x) = \frac{\sqrt[3]{x+4}}{2}$

$2g(x) = \sqrt[3]{x+4}$

$(2g(x))^3 = x+4$

$(2g(x))^3 - 4 = x$

$(2x)^3 - 4 = g^{-1}(x)$

$g^{-1}(g(x)) = \left(\frac{2 \cdot \sqrt[3]{x+4}}{2}\right)^3 - 4$

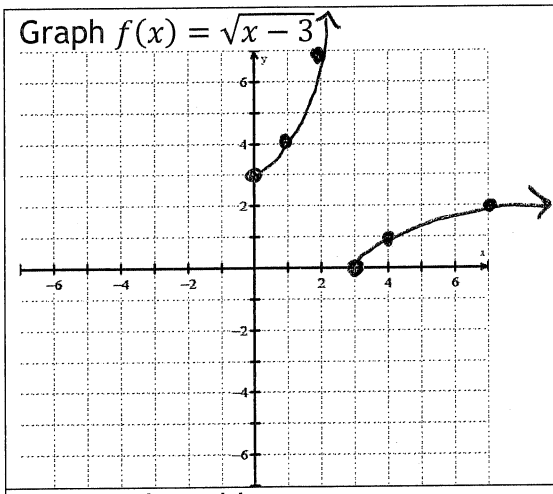
$= (\sqrt[3]{x+4})^3 - 4$

$= x + 4 - 4$

$= x$

9.

Graph $f(x) = \sqrt{x-3}$



Find the inverse.

$f(x) = \sqrt{x-3}$

$(f(x))^2 = x - 3$

$(f(x))^2 + 3 = x$

$(x)^2 + 3 = f^{-1}(x)$

for domain $0 \le x < \infty$

Fill out the table:

	Original	Inverse
Domain	$3 \le x < \infty$	$0 \le x < \infty$
Range	$0 \le y < \infty$	$3 \le y < \infty$
x-intercept	$(3,0)$	none
y-intercept	none	$(0,3)$

Why isn't the inverse a full parabola with a domain of $(-\infty, \infty)$?

- it wouldn't match with the original
- domain & range for inverse & original should switch

10.

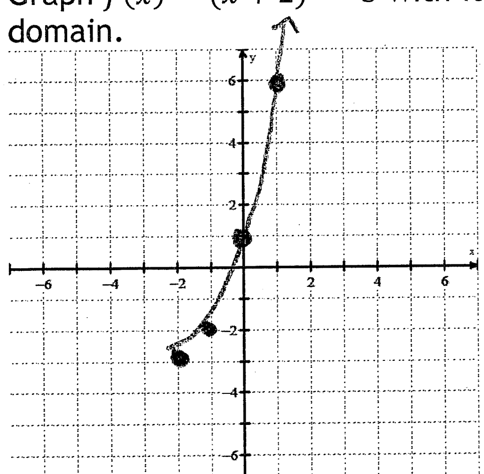
Think about the function $f(x) = (x+2)^2 - 3$. Will its inverse be a function? Why not?

no, sideways parabola would not be a function

Fill out the table:

	Original	Inverse
Domain	$-2 \le x < \infty$	$-3 \le x < \infty$
Range	$-3 \le y < \infty$	$-2 \le y < \infty$
x-intercept	$\approx (-0.5, 0)$	$(1, 0)$
y-intercept	$(0, 1)$	$\approx (0, -0.5)$

Graph $f(x) = (x+2)^2 - 3$ with its restricted domain.



Find the inverse.

$f(x) = (x+2)^2 - 3$

$f(x) + 3 = (x+2)^2$

$\pm\sqrt{f(x)+3} = x + 2$

$\pm\sqrt{f(x)+3} - 2 = x$

only want positive sq. rt.

$\sqrt{x+3} - 2 = f(x)$