CCSS Advanced Algebra 3 AA4: Transformations Review Name________________

Part 1: Write the expression that fits each blank. Then name the transformation(s).

$f(x) = x^2$ $g(x) = |x|$ $h(x) = \sqrt{x}$ $j(x) = x^3$ $k(x) = \sqrt[3]{x}$

Expression	$f(x+2) =$ ________	$2g(x) =$ ________	$h(x) - 4 =$ ________	$j(0.1x) =$ ________
Transformation				
Expression:	$2k(x-1) =$ ________	$g(2x)+4 =$ ________	$f(2(x-5)) =$ ________	$4h(x)+3 =$ ________
Transformation				

Part 2: Write the equation for each function described below:

1. Parent *Quadratic function* ($y = x^2$) is reflected over the x-axis, translated down 4 units and left 2 units.

2. Parent *Cubic function* ($y = x^3$) is stretched vertically by a factor of 3, translated right 5 units and up 1 unit.

3. Parent *Square Root function* ($y = \sqrt{x}$) is reflected over the y-axis, compressed vertically by a factor of $\frac{1}{2}$ and translated left 4 units.

4. Parent *Cube Root function* ($y = \sqrt[3]{x}$) is reflected over the y-axis, compressed horizontally by a factor of 8 and translated up 3.

5. Parent *Absolute Value function* ($y = |x|$) is stretched vertically by a factor of 2, translated right 3 units and reflected over the x-axis.

6. Parent *Linear function* ($y = x$) is reflected over the x-axis, stretched vertically by a factor of 4 and translated right 2 units.

Part 3: Find the exact equation of each function described below.

1. Parent *quadratic function* with a vertex of (2,-3) that passes through the point (3,12)

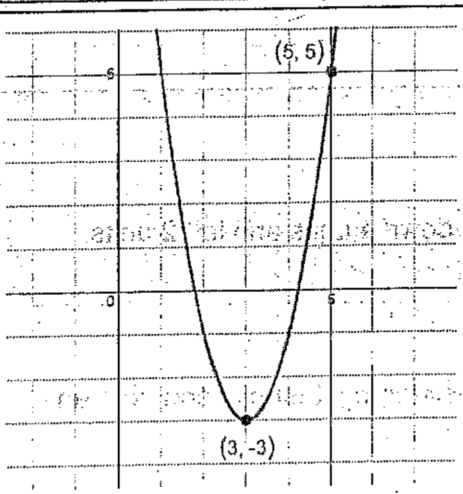
2. Parent *cubic function* with an inflection point of (-4, -3) that passes through the point (-5,2)

3. Parent *square root* function with a vertex of (3,5) that passes through the point (7,-3)

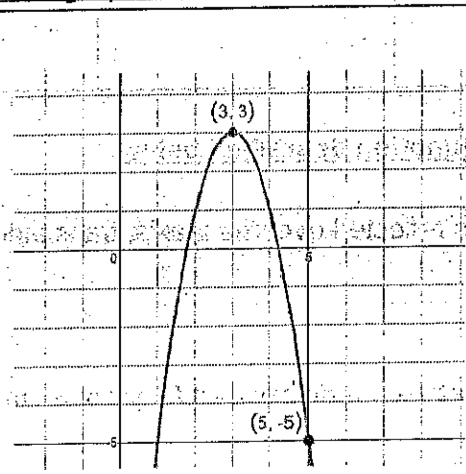
4. Parent *cube root* function with an inflection point of (-1,-1) that passes through the origin

5. Parent *absolute value* function with a vertex of (7, -3) that passes through the origin

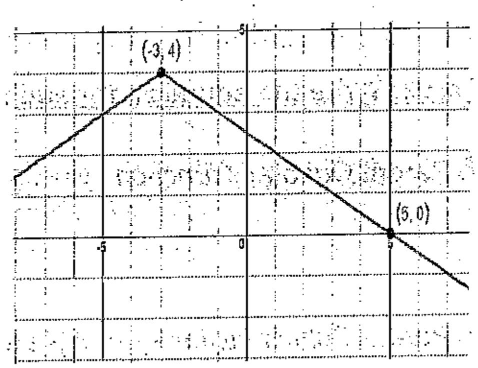
Part 4: Find the exact equation of each graph below:



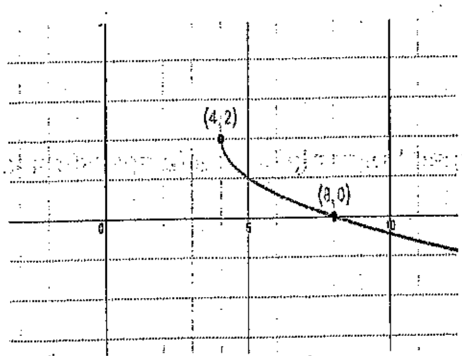
Equation:



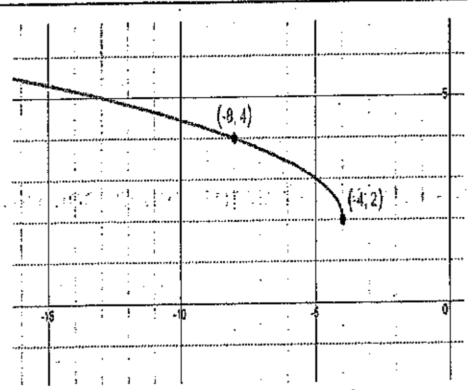
Equation:



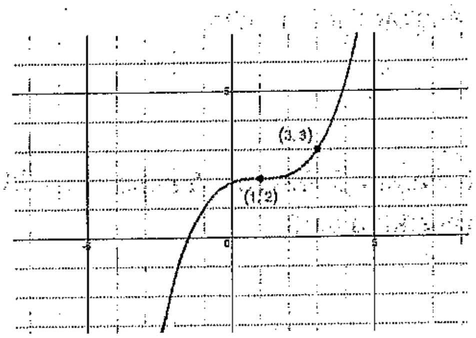
Equation:



Equation:



Equation:



Equation:

Finding Dilation Factors

IF NEEDED: Watch this screencast for more information.

VERTICAL DILATIONS

1. A Quadratic Function is represented by the function $f(x) = a(x-2)^2 + 3$.
   a. Explain why the vertex of this parabola is $(2,3)$.

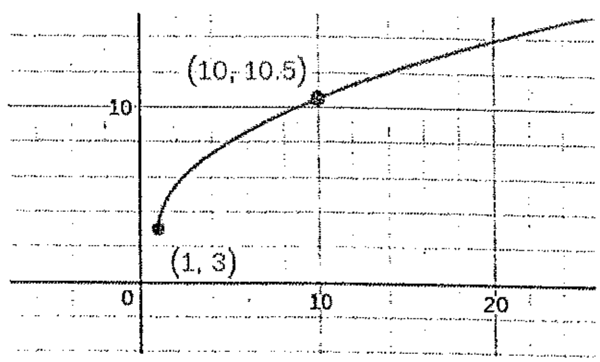
   b. The parabola has an y-intercept at $(0,\ 15)$. Explain why the following equation must be true: $15 = a(0-2)^2 + 3$.

   c. Solve the equation in part (b) for a and write the completed function $f(x)$.

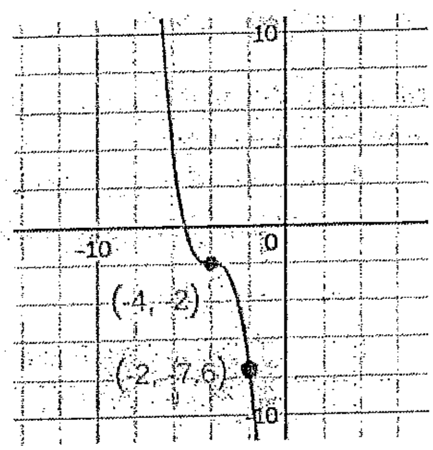
2. An Absolute Value function has a vertex at $(5,\ -10)$.
   a. Replace the #s to write the function in the form $g(x) = a|x-\#| + \#$.

   b. If $g(10) = 0$, find the value of a and write the completed function.

3. A Square Root function is shown. Use the technique from the previous questions to find a and write the function in the form $h(x) = a\sqrt{x-\#} + \#$.

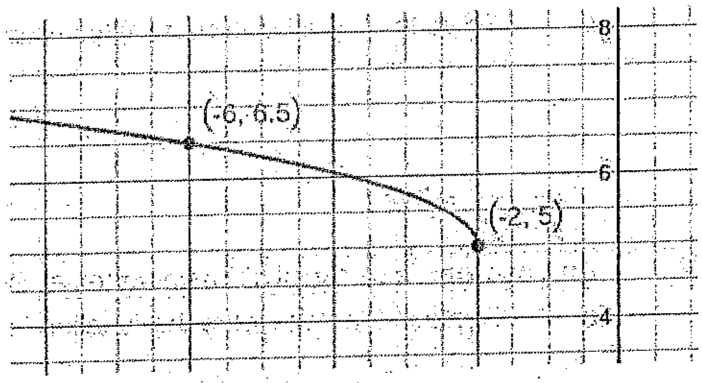


4. A Cubic function is shown. Use the technique from the previous questions to find a and write the function in the form $p(x) = a(x - \#)^3 + \#$.



HORIZONTAL DILATIONS

5. Another Square Root function is shown. Use the techniques from questions 1 & 2 to find b and write the function in the form $k(x) = \sqrt{b(x - \#)} + \#$.



Practice: Find the equation of each function described below.

a. **Quadratic. Vertex at (2,3). Passes through (3, 7)**

b. **Absolute Value. Vertex at (-3,-5). Passes through (5,10)**

c. **Square Root. Vertex at (5,10). Passes through (7,20)**

d. **Square Root. Vertex at (5,10). Passes through (-7,-20)**

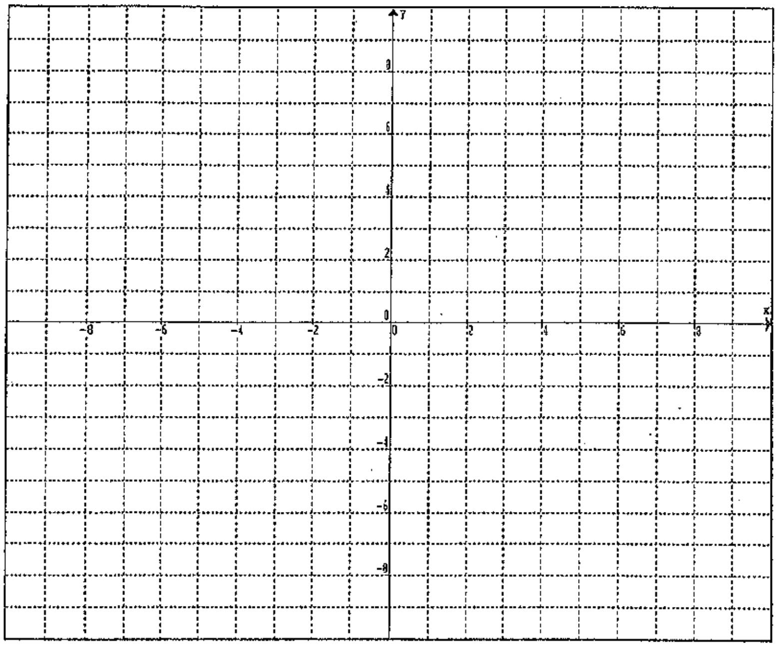
Graphing Quadratic Functions

1. $f(x) = x^2$

Vertex =

y-intercept :

x-intercept:

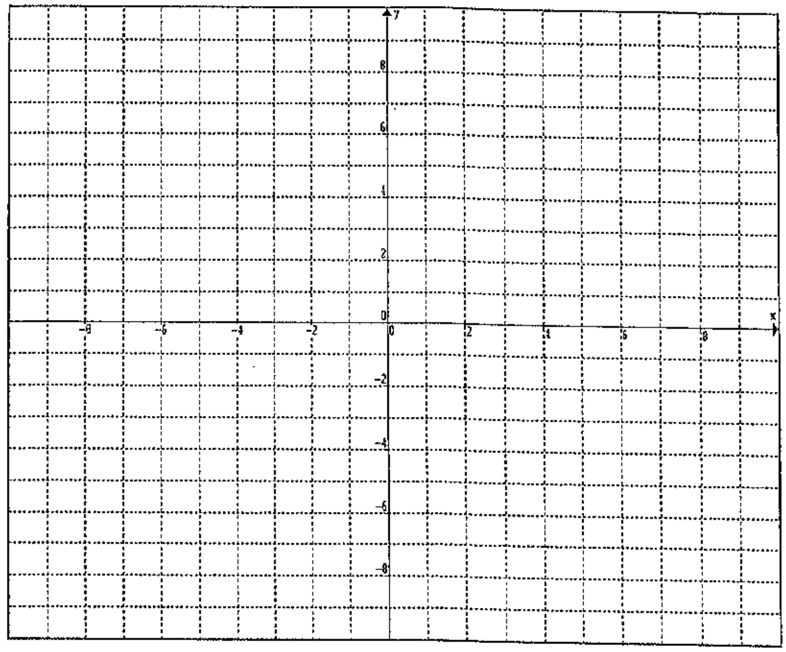


2. $f(x) = x^2 + 5$

Vertex =

y-intercept :

x-intercept:

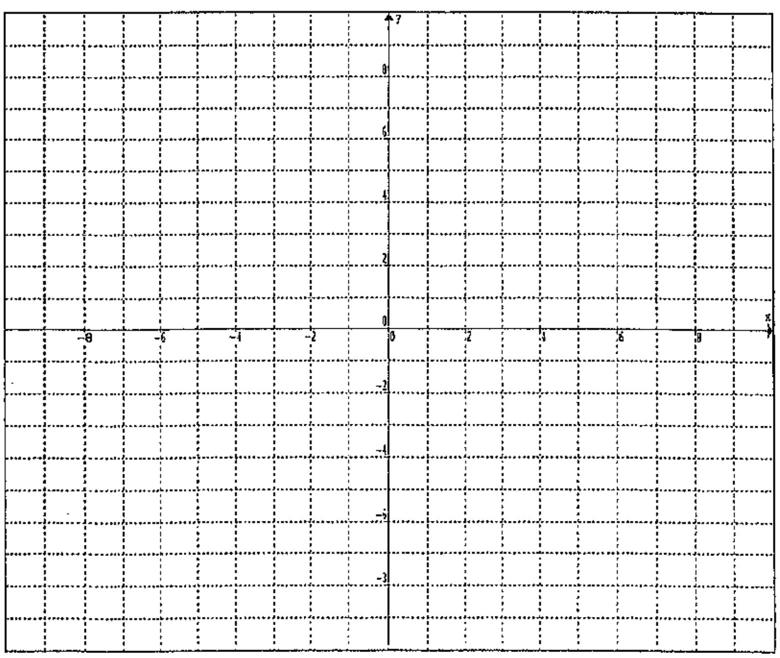


3. $f(x) = (x+3)^2$

Vertex =

y-intercept :

x-intercept:

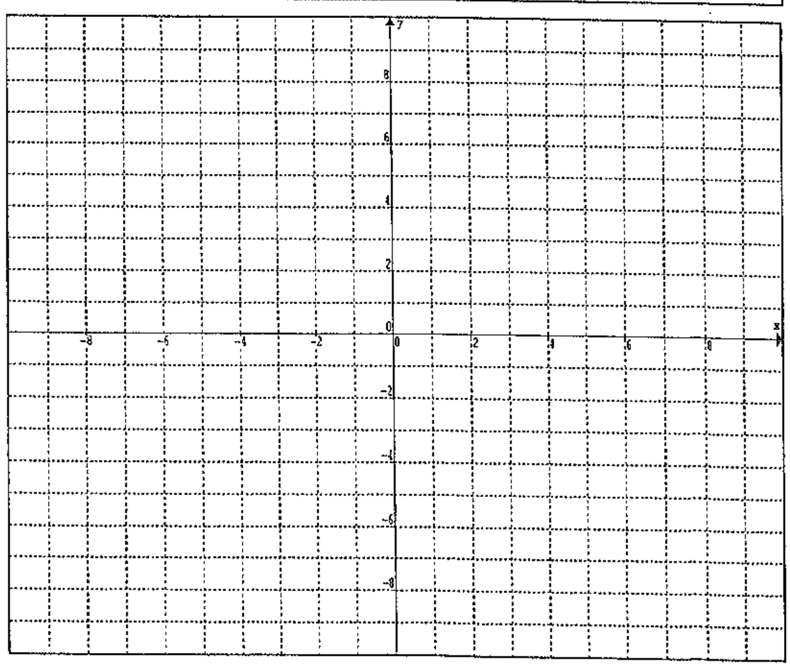


4. $f(x) = (x-4)^2 - 3$

Vertex =

y-intercept :

x-intercept:

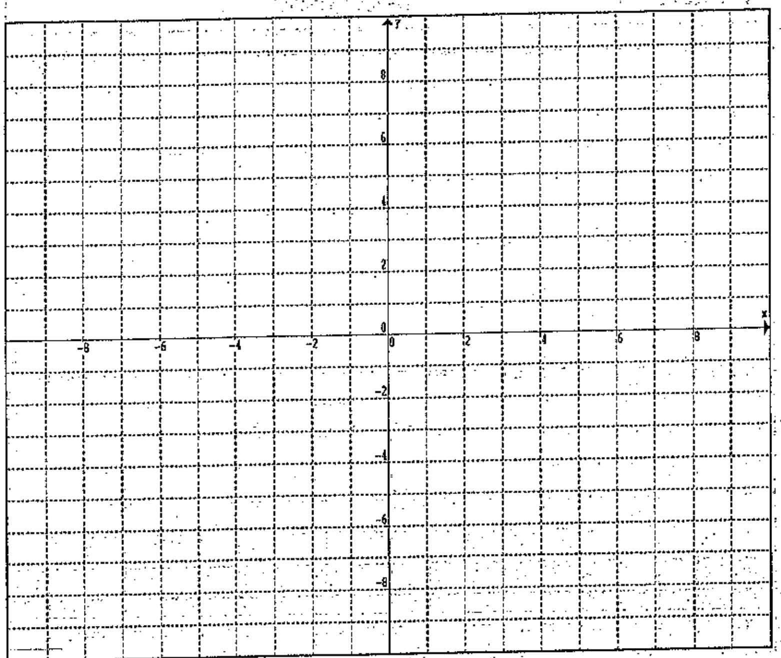


5. $f(x) = -x^2$

Vertex =

y-intercept :

x-intercept:

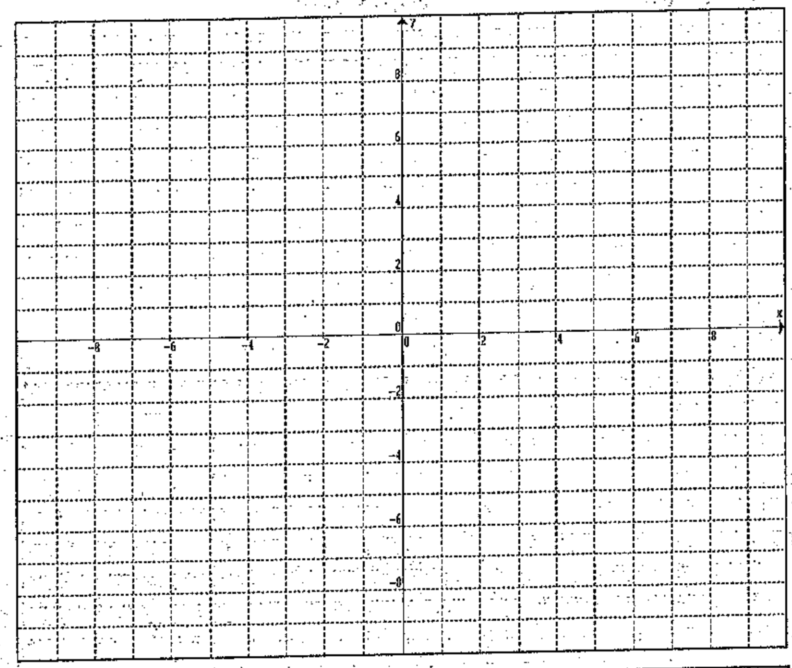


6. $f(x) = -x^2 + 4$

Vertex =

y-intercept :

x-intercept:

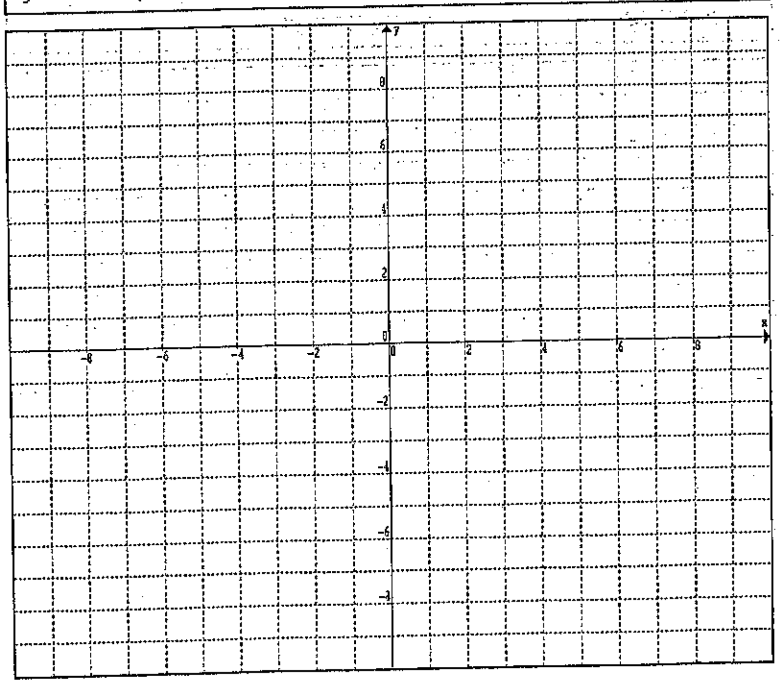


7. $f(x) = -(x+3)^2$

Vertex =

y-intercept :

x-intercept:

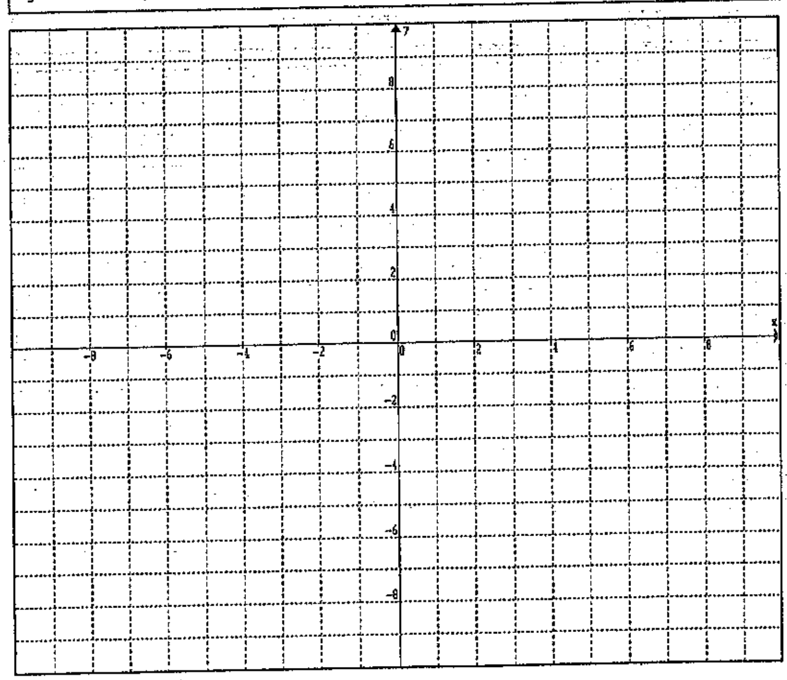


8. $f(x) = -(x-1)^2 - 3$

Vertex =

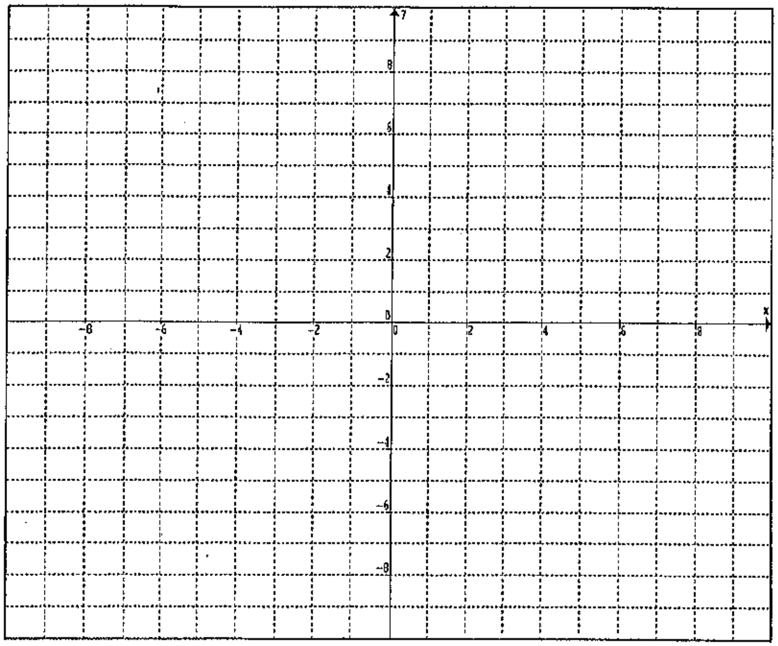
y-intercept :

x-intercept:



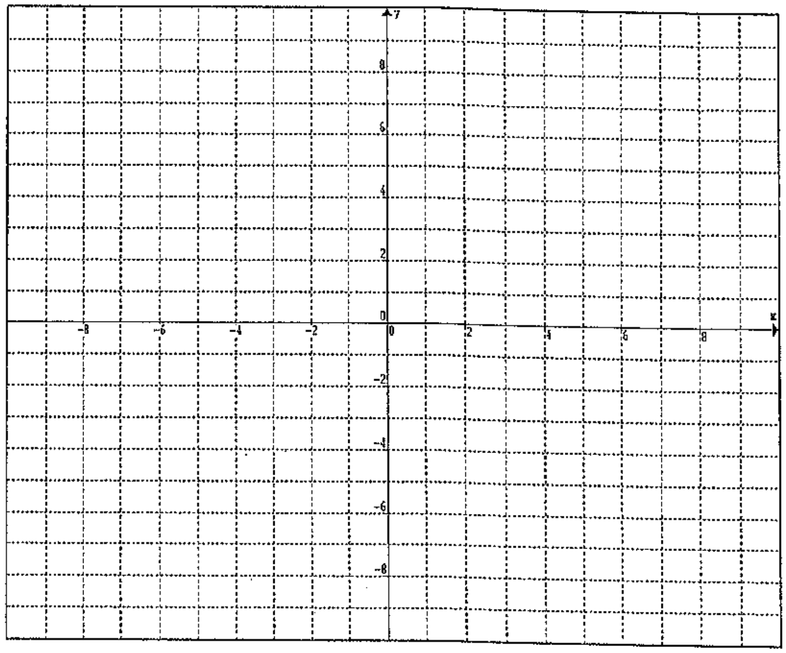
9. $f(x) = 2x^2$

Vertex =
y-intercept : x-intercept:



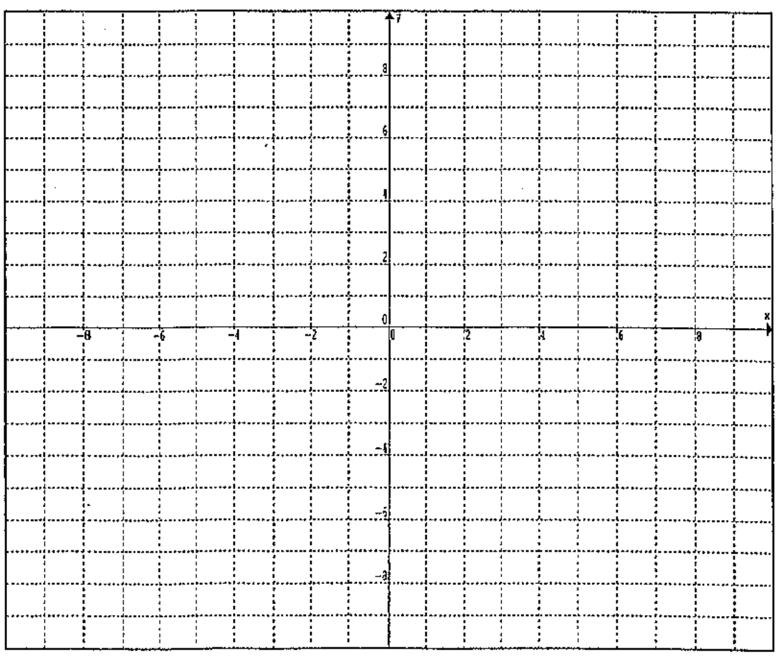
10. $f(x) = -2x^2$

Vertex =
y-intercept : x-intercept:



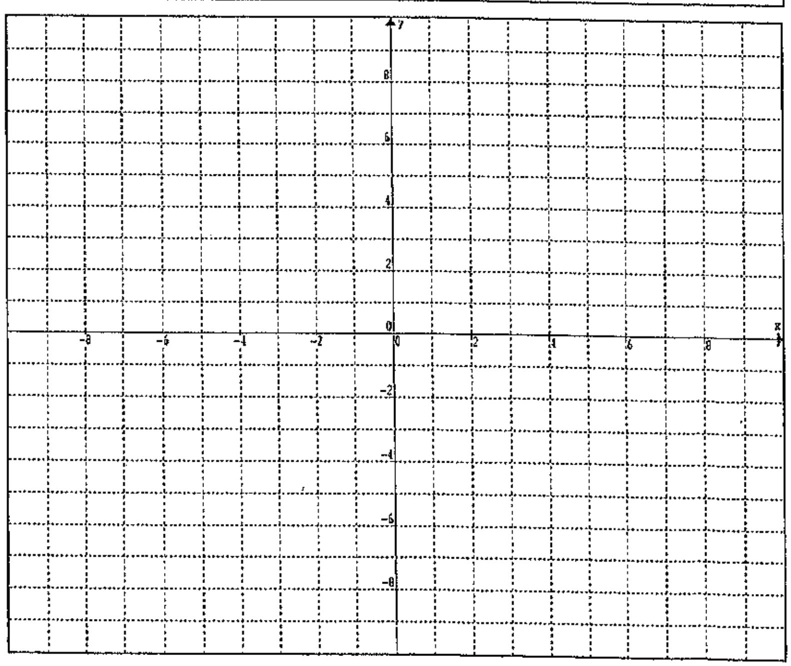
11. $f(x) = 2(x+3)^2 - 6$

Vertex =
y-intercept : x-intercept:



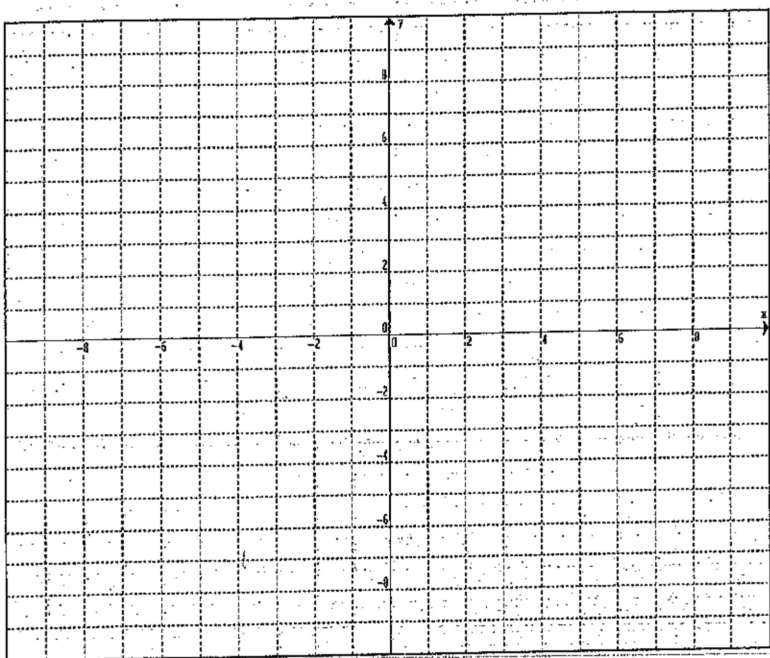
12. $f(x) = -2(x-1)^2 - 2$

Vertex =
y-intercept : x-intercept:



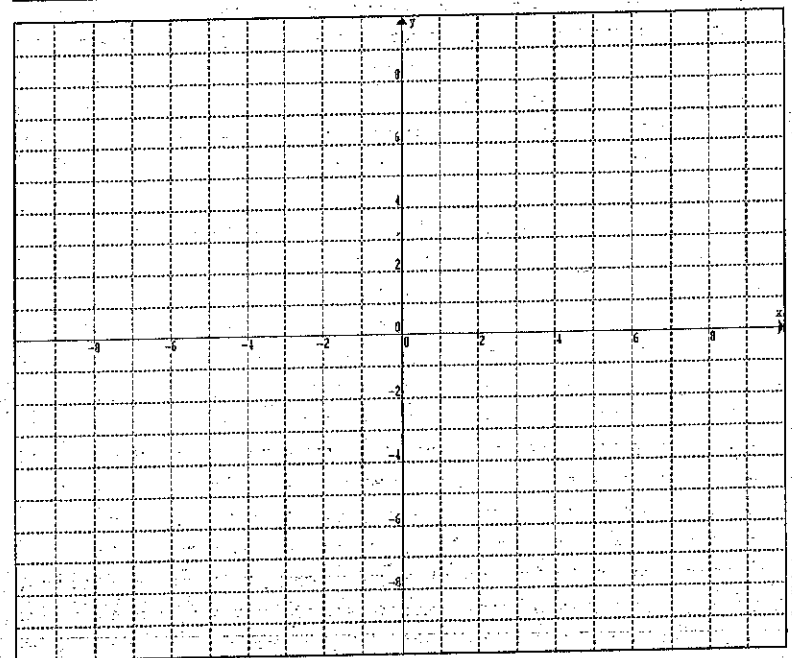
13. $f(x)=x^2+4x-5$

Vertex =
y-intercept :
x-intercept:



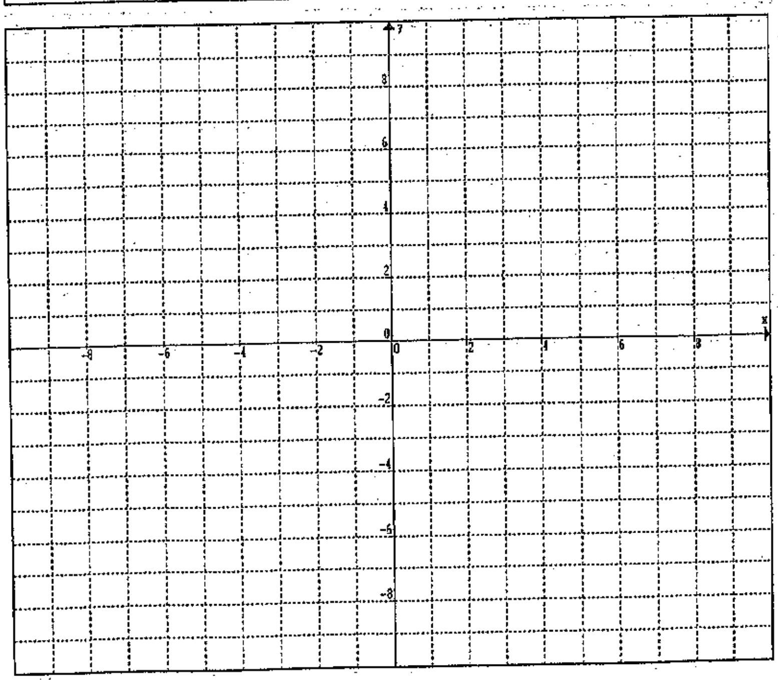
14. $f(x)=x^2+6x+2$

Vertex =
y-intercept :
x-intercept:



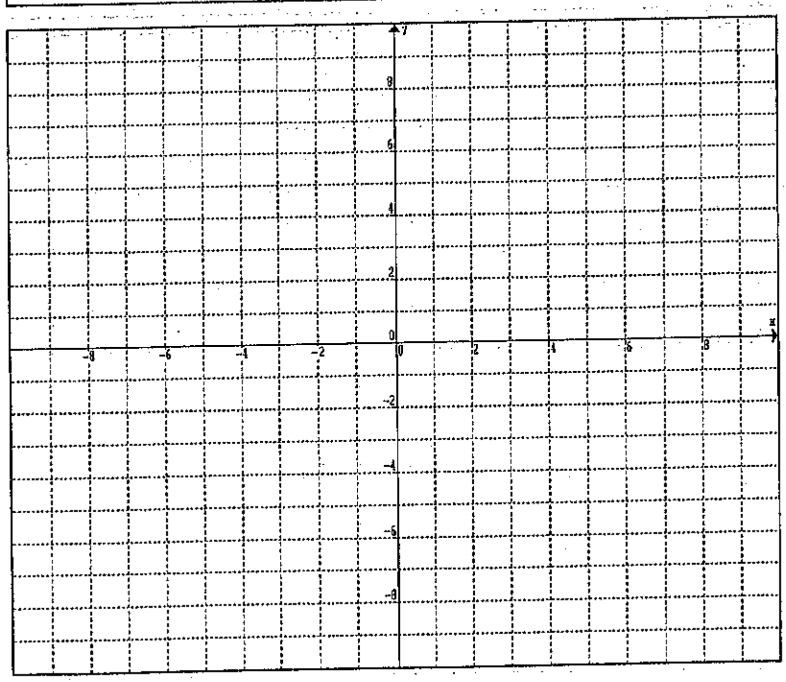
15. $f(x)=x^2+4x+7$

Vertex =
y-intercept :
x-intercept:



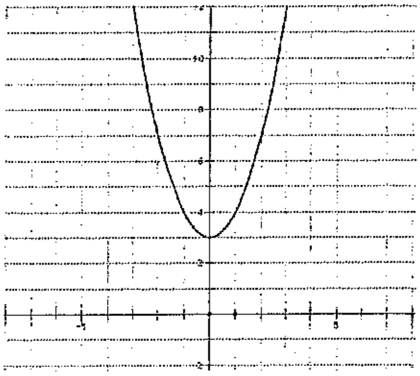
16. $f(x)=x^2-6x+2$

Vertex =
y-intercept :
x-intercept:

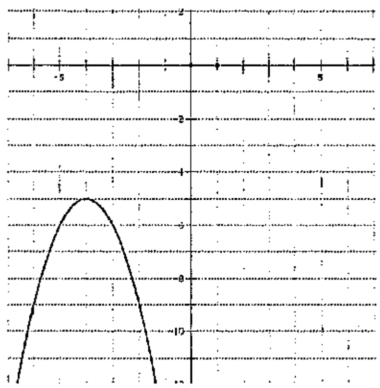


Write an equation of each graph below in the form $f(x) = a(x-h)^2 + k$.

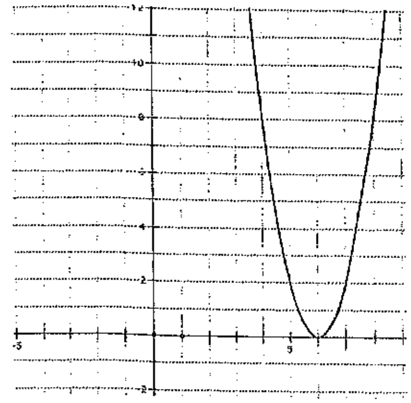
33. $f(x) =$ ____________________



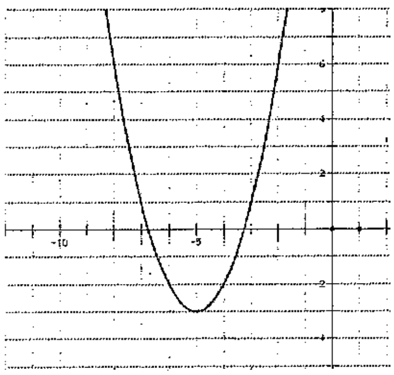
34. $f(x) =$ ____________________



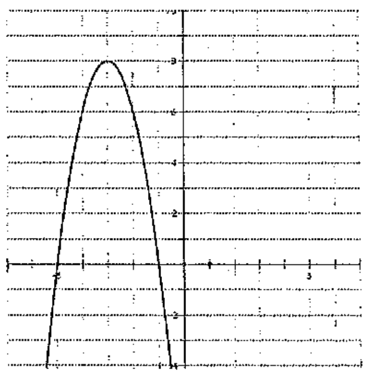
35. $f(x) =$ ____________________



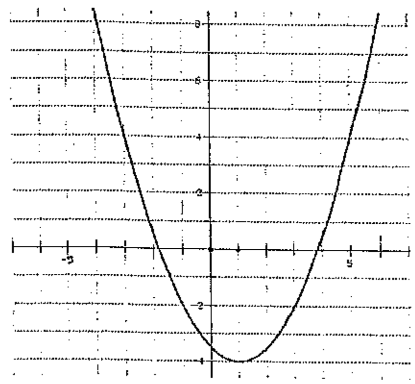
36. $f(x) =$ ____________________



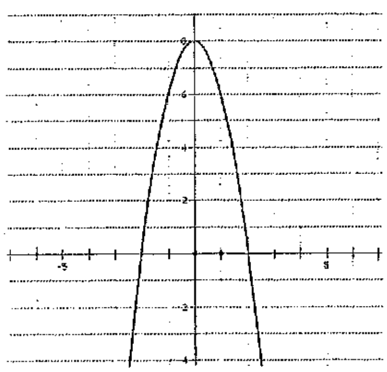
37. $f(x) =$ ____________________



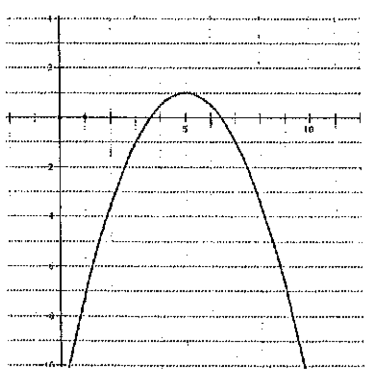
38. $f(x) =$ ____________________



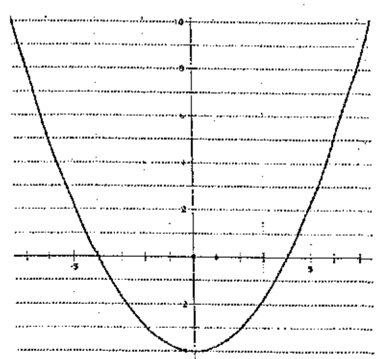
39. $f(x) =$ ____________________



40. $f(x) =$ ____________________



41. $f(x) =$ ____________________



1. Identify each transformation (or transformations) below. Be specific.

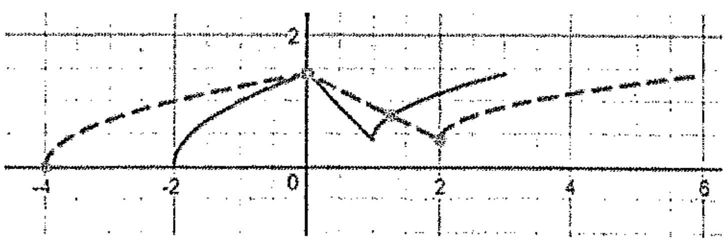
Transformations:		
HORIZONTAL TRANSLATION (Left or Right)	VERTICAL TRANSLATION (Up or Down)	VERTICAL REFLECTION
HORIZONTAL REFLECTION	HORIZONTAL DILATION (Stretch or Compress)	VERTICAL DILATION (Stretch or Compress)

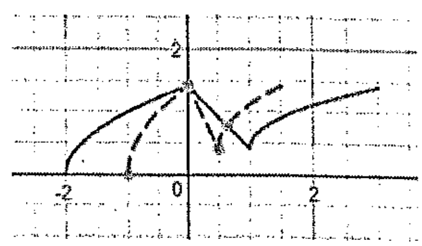
a. $f(x)+10$ b. $f(x-3)$ c. $f(x+8)$

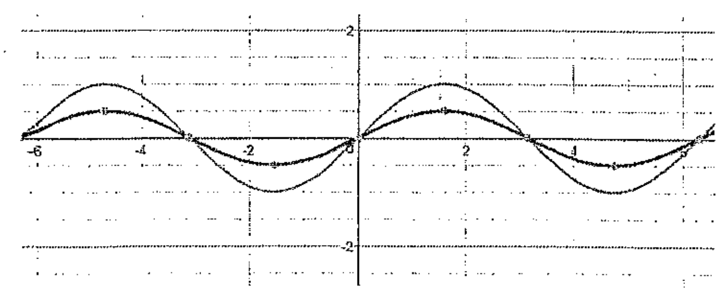
d. $3f(x)$ e. $-f(x)$ f. $f(0.5x)$

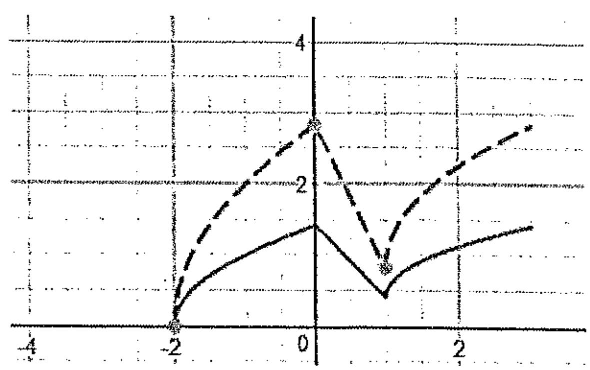
g. $f(-x)$ h. $f(2(x-1))$ i. $f(x+3)+3$

2. Which image(s) below show a horizontal dilation and which show(s) a vertical dilation? How can you tell?









3. Let the Parent LINEAR Function be $g(x) = x$.

a. Explain GRAPHICALLY why a vertical translation up 1 unit results in the same function as a horizontal translation left 1 unit.

b. Will a VERTICAL REFLECTION of $g(x) = x$ look differently than a HORIZONTAL REFLECTION of $g(x) = x$? Explain how you know.

c. Is $h(x) = 3x$ a VERTICAL or HORIZONTAL DILATION of $g(x) = x$? Explain how you know.

4. Consider the Quadratic Function $n(x) = x^2 + 10x + 21$.

a. Factor to show that $n(x) = (x+\#)(x+\#)$.

b. The VERTEX is halfway between the x-intercepts. Find the x- and y-coordinates of the vertex.

c. What transformation(s) on $f(x) = x^2$ result in $n(x)$? Be specific.

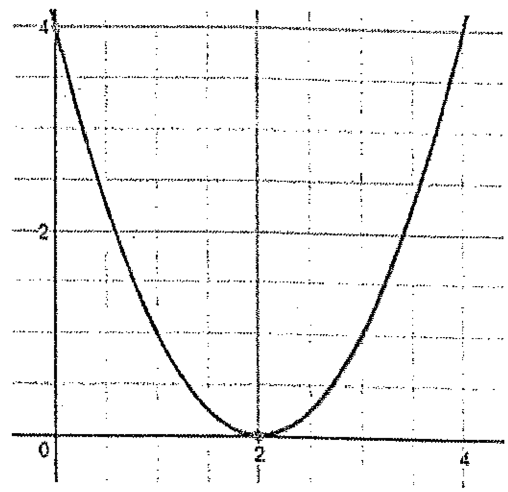
d. Evaluate $n(0)$. What does $n(0)$ tell you about the GRAPH of $n(x)$?

e. Find the VERTEX of $n(x+1) - 3$.

1. What is the difference between the graphs of $f(x) = x^2 + 1$ and $g(x) = x^2 - 1$?

2. What is the difference between the graphs of $f(x) = x^2 + 1$ and $h(x) = -x^2 + 1$?

3. Linsey wants to create a design on desmos and started with the parabola shown. What equation did she use to create this parabola?



4. Without graphing, describe the differences in the parabolas that represent the function $p(x) = 4x^2$ and $q(x) = 0.25x^2$.

Vocabulary:

Parent Graph: the graphical representation of the most basic form of function family. The parent graph for Quadratic Functions is the graph of $y = x^2$.

Transformation: changes to the shape, orientation or location of the parent graph. There are three main types of transformations that we will study -- *Translation, Dilation and Reflection* (in Geometry, you also explored *Rotation*).

- Translation (Slide): moving all points on a graph horizontally or vertically a fixed amount.
  - Notation:
    - Vertical Translation $f(x)+k$
    - Horizontal Translation $f(x-k)$

- Dilation (Stretch or Compression): an increase or decrease in the height (vertical dilation) or width (horizontal dilation) of a graph by a factor. For example, $y = 3x^2$ vertically stretches the parent function by a factor of 3.
  - Notation:
    - Vertical Dilation $k\,f(x)$
    - Horizontal Dilation $f(kx)$

- Reflection (Flip): mirroring a graph over a fixed line (typically the x-axis or y-axis).
  - Notation:
    - Vertical Reflection $-f(x)$
    - Horizontal Reflection $f(-x)$

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7. For each function below, describe how the parent function $f(x) = x^2$ was transformed. The first one is done as an example.

a. $y = x^2 - 3$ the parent function was translated 3 units down.

b. $y = x^2 + 10$

c. $y = (x-4)^2$

d. $y = 0.5x^2$

e. $y = (2x)^2$

f. $y = -(x-1)^2$ (describe both transformations)

g. $y = 5(x+2)^2 - 5$ (describe all transformations)

CCSS Advanced Algebra 3 More Quadratic Transformations:

1. Let $f(x) = (x+7)(x-5)$.
   a. What are the x-intercepts of the function? y-intercept?
   b. Consider the transformation $g(x) = f(x-2)$. What are the x-intercepts of $g(x)$?
   c. Consider the transformation $h(x) = 3f(x) - 1$. What is the y-intercept of $h(x)$?

2. For the function $w(x) = x^2 + 6x + 8$,
   d. Find the x-intercepts and the y-intercept.
   e. What transformation would be applied to $w(x)$ that would result in the new function $v(x) = (x+5)(x+7)$?
   f. Explain why the transformed function $u(x) = 5w(x)$ has the same x-intercepts as $w(x)$.

3. Write $f(x) = (x+7)(x-5)$
   a. in standard form, $f(x) = ax^2 + bx + c$.
   b. Complete the square (see below for notes) to write $f(x)$ in Graphing Form $f(x) = a(x-h)^2 + k$ and write the vertex of the parabola.
   c. What is the vertex of $g(x) = f(x-2)$?
   d. What is the vertex of $h(x) = 3f(x) - 1$?

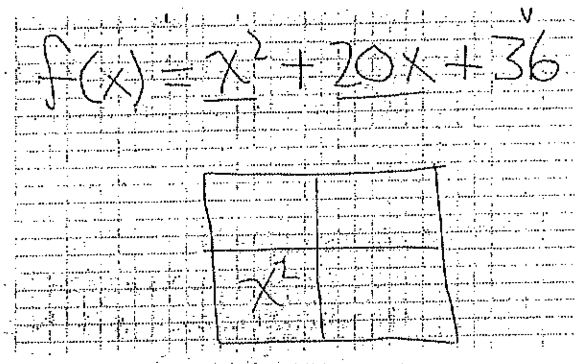
4. Completing the square practice (see below for notes). Write each quadratic function in Graphing Form and determine the vertex (BOTH x AND y):
   a. $p(x) = x^2 + 10x - 24$
   b. $q(x) = x^2 - 5x + 6$
   c. $r(x) = 2x^2 + 6x - 36$
   d. $s(x) = 5x^2 - 20x + 25$
   e. $t(x) = 10x^2 - 18x - 36$

5. For each quadratic equation above, describe the transformations that would be required to go from the parent graph ($y = x^2$) to the new function. Be specific using the terms horizontal/vertical translation, reflection, dilation.

Completing the Square Notes (Converting a Quadratic Function from Standard to Graphing Form):

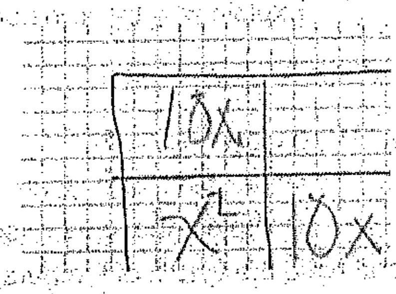
Example: $f(x) = x^2 + 20x + 36$

Step 1: Create a generic rectangle and put the x^2 in the lower left corner.

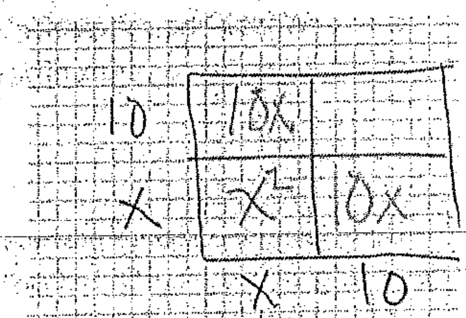


Step 2: Split the $20x$ in half and place each half in the generic rectangle.

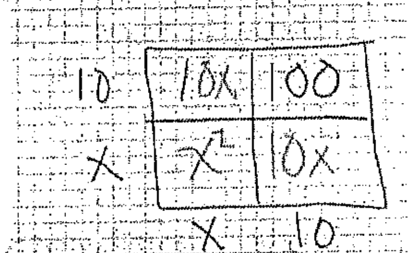
**Why does it make sense to do this?



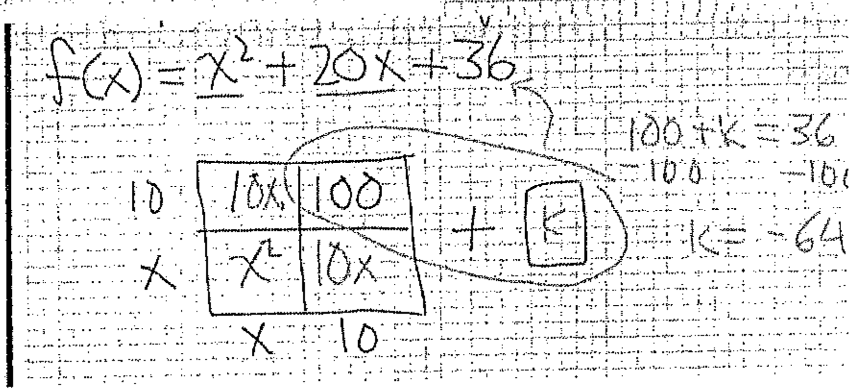
Step 3: Fill out the outside (base and height) of the generic rectangle.



Step 4: Complete the inside of the generic rectangle using the outside values.



Step 5: Determine what value must be added to the generic rectangle to match the original function.



Step 5: Write the function in Graphing Form.

$$f(x) = (x + 10)^2 - 64$$