CCSS Advanced Algebra 3 AA4: Transformations Review Name________________________

Part 1: Write the expression that fits each blank. Then name the transformation(s).

$f(x) = x^2$ $g(x) = |x|$ $h(x) = \sqrt{x}$ $j(x) = x^3$ $k(x) = \sqrt[3]{x}$

Expression	$f(x+2) = (x+2)^2$	$2g(x) = 2\|x\|$	$h(x) - 4 = \sqrt{x} - 4$	$j(0.1x) = (0.1x)^3$
Transformation	Left 2	Twice as tall	Down 4	10 times wider
Expression:	$2k(x-1) = 2\sqrt[3]{x-1}$	$g(2x) + 4 = \|2x\|+4$	$f(2(x-5)) = (2(x-5))^2$	$4h(x) + 3 = 4\sqrt{x} + 3$
Transformation	Right 1, twice as tall	Up 4 Twice as narrow	5 Right Twice as narrow	Up 3 4 times as tall

Part 2: Write the equation for each function described below:

1. Parent *Quadratic function* ($y = x^2$) is reflected over the x-axis, translated down 4 units and left 2 units.

$y = -(x+2)^2 - 4$

2. Parent *Cubic function* ($y = x^3$) is stretched vertically by a factor of 3, translated right 5 units and up 1 unit.

$y = 3(x-5)^3 + 1$

3. Parent *Square Root function* ($y = \sqrt{x}$) is reflected over the y-axis, compressed vertically by a factor of $\frac{1}{2}$ and translated left 4 units.

$y = \frac{1}{2}\sqrt{-x+4}$

4. Parent *Cube Root function* ($y = \sqrt[3]{x}$) is reflected over the y-axis, compressed horizontally by a factor of 8 and translated up 3.

$y = \sqrt[3]{-8x} + 3$

5. Parent *Absolute Value function* ($y = |x|$) is stretched vertically by a factor of 2, translated right 3 units and reflected over the x-axis.

$y = -2|x-3|$

6. Parent *Linear function* ($y = x$) is reflected over the x-axis, stretched vertically by a factor of 4 and translated right 2 units.

$y = -4(x-2)$

Part 3: Find the exact equation of each function described below.

1. Parent *quadratic function* with a vertex of (2,-3) that passes through the point (3,12)

$y=a(x-2)^2-3$ → $12=a(3-2)^2-3$ → $12=a(1)^2-3$ → $15=a$ → $y=15(x-2)^2-3$

2. Parent *cubic function* with an inflection point of (-4, -3) that passes through the point (-5,2)

$y=a(x+4)^3-3$ → $2=a(-5+4)^3-3$ → $2=a(-1)^3-3$ → $5=a(-1)$ → $a=-5$ → $y=-5(x+4)^3-3$

3. Parent *square root* function with a vertex of (3,5) that passes through the point (7,-3)

$y=a\sqrt{x-3}+5$ → $-3=a\sqrt{7-3}+5$ → $-3=a\sqrt{4}+5$, -5, -5, $-8=a\cdot 2$ → $-4=a$ → $y=-4\sqrt{x-3}+5$

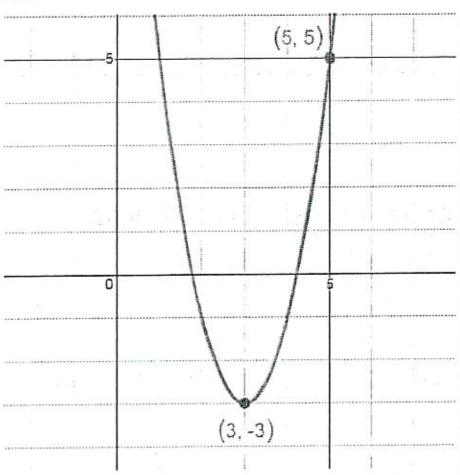
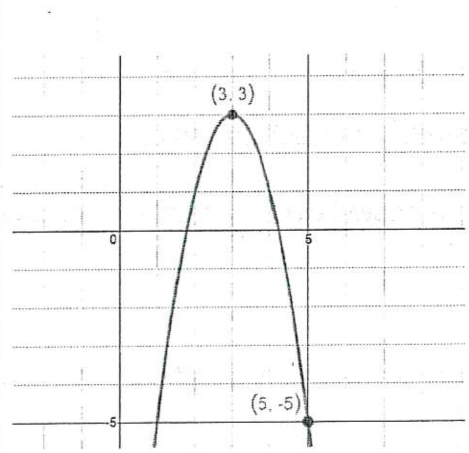
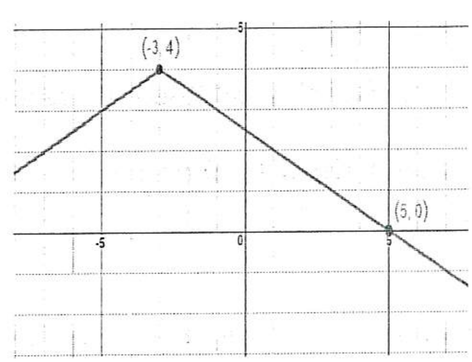
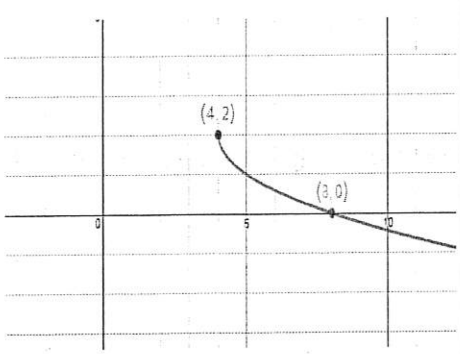
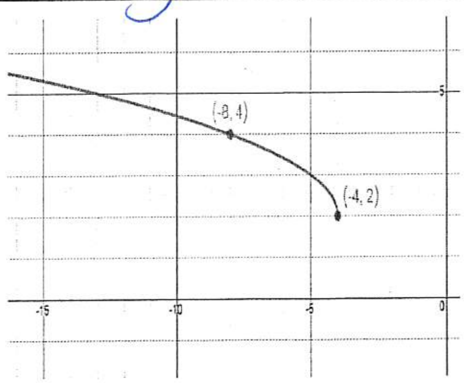
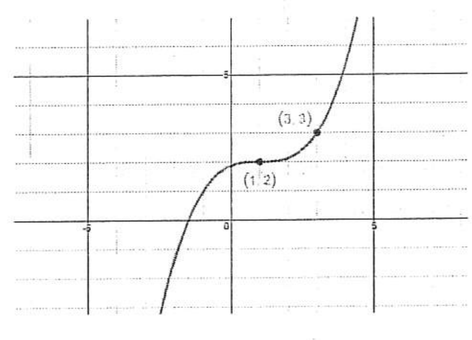
4. Parent *cube root* function with an inflection point of (-1,-1) that passes through the origin

$y=a\sqrt[3]{x+1}-1$ → $0=a\sqrt[3]{0+1}-1$ → $0=a\sqrt[3]{1}-1$ → $1=a\cdot 1$ → $y=\sqrt[3]{x+1}-1$

5. Parent *absolute value* function with a vertex of (7, -3) that passes through the origin

$y=a|x-7|-3$ → $0=a|0-7|-3$ → $0=a(7)-3$ → $3=a(7)$ → $a=3/7$ → $y=\frac{3}{7}|x-7|-3$

Part 4: Find the exact equation of each graph below:

		
Equation: $y=2(x-3)^2-3$	Equation: $y=-2(x-3)^2+3$	Equation: $y=-\frac{1}{2}\lvert x+3\rvert+4$
		
Equation: $y=-\sqrt{x-4}+2$	Equation: $y=\sqrt{-(x+4)}+2$	Equation: $y=\frac{1}{8}(x-1)^3+2$

CCSS Advanced Algebra 3 Unit 4 Transformations Name ______________

Finding Dilation Factors

IF NEEDED: Watch this screencast for more information.

VERTICAL DILATIONS

1. A Quadratic Function is represented by the function $f(x) = a(x-2)^2 + 3$.

   a. Explain why the vertex of this parabola is $(2,3)$.

   Because $2-2=0$ & 0 is the smallest possible value for a quadratic.

   b. The parabola has an y-intercept at $(0,\ 15)$. Explain why the following equation must be true: $15 = a(0-2)^2 + 3$.

   Because you plugged in $x=0$ & $y=15$

   c. Solve the equation in part (b) for a and write the completed function $f(x)$.

   $15 = a(0-2)^2+3$

   $15 = a(-2)^2+3$

   $15 = a \cdot 4 + 3$

   $-3 \qquad -3$

   $\frac{12}{4} = \frac{a \cdot 4}{4}$

   $3 = a$

   $y = 3(x-2)^2+3$

2. An Absolute Value function has a vertex at $(5,\ -10)$.

   a. Replace the #s to write the function in the form $g(x) = a|x-\#|+\#$.

   $g(x) = a|x-5|-10$

   b. If $g(10) = 0$, find the value of a and write the completed function.

   $0 = a|10-5|-10$

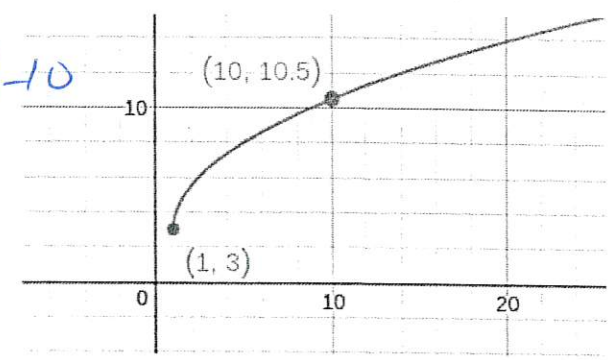
   $+10 \qquad +10$

   $\frac{10}{5} = \frac{a|5|}{5}$

   $2 = a$

   $g(x) = 2|x-5|-10$

3. A Square Root function is shown. Use the technique from the previous questions to find a and write the function in the form $h(x) = a\sqrt{x-\#}+\#$.



$h(x) = a\sqrt{x-1}+3$

$10.5 = a\sqrt{10-1}+3$

$10.5 = a\sqrt{9}+3$

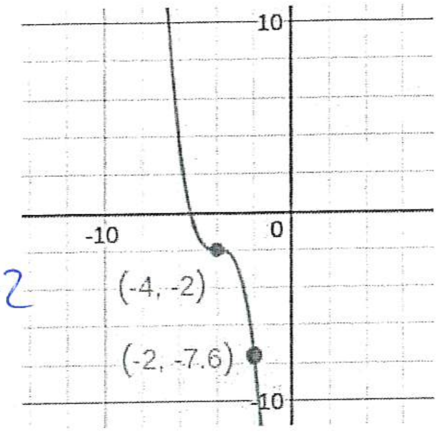
$-3 \qquad -3$

$\frac{7.5}{3} = \frac{a \cdot 3}{3}$

$2.5 = a$

$h(x) = 2.5\sqrt{x-1}+3$

4. A Cubic function is shown. Use the technique from the previous questions to find a and write the function in the form $p(x) = a(x-\#)^3 + \#$.



$y = a(x+4)^3 - 2$

$-7.6 = a(-2+4)^3 - 2$

$-7.6 = a(2)^3 - 2$

$+8 \quad +2$

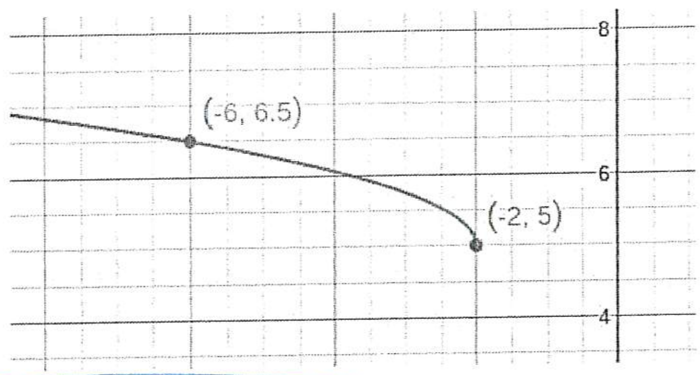
$-5.6 = a \cdot 8$

$\frac{-5.6}{8} = \frac{a \cdot 8}{8}$

$y = -\frac{5.6}{8}(x+4)^3 - 2$

HORIZONTAL DILATIONS

5. Another Square Root function is shown. Use the techniques from questions 1 & 2 to find b and write the function in the form $k(x) = \sqrt{b(x-\#)} + \#$.



$K(x) = \sqrt{b(x+2)} + 5$

$6.5 = \sqrt{b(-6+2)} + 5$

$6.5 = \sqrt{b(-4)} + 5$

$-5 \quad -5$

$1.5^2 = \sqrt{b(-4)}^2$

$2.25 = b(-4)$

$2.25/-4 = b$

$K(x) = \sqrt{\frac{2.25}{-4}(x+2)} + 5$

Practice: Find the equation of each function described below.

a. **Quadratic. Vertex at (2,3). Passes through (3, 7)**

$y = a(x-2)^2 + 3$

$7 = a(3-2)^2 + 3$

$7 = a(1) + 3$

$a = 4$

$y = 4(x-2)^2 + 3$

b. **Absolute Value. Vertex at (-3,-5). Passes through (5,10)**

$y = a|x+3| - 5$

$10 = a|5+3| - 5$

$10 = a|8| - 5$

$\frac{15}{8} = \frac{a \cdot 8}{8}$

$15/8 = a$

$y = \frac{15}{8}|x+3| - 5$

c. **Square Root. Vertex at (5,10). Passes through (7,20)**

$y = a\sqrt{x-5} + 10$

$20 = a\sqrt{7-5} + 10$

$-10 \quad -10$

$\frac{10}{\sqrt{2}} = \frac{a\sqrt{2}}{\sqrt{2}}$

$y = \frac{10}{\sqrt{2}}\sqrt{x-5} + 10$

d. **Square Root. Vertex at (5,10). Passes through (-7,-20)**

$y = \sqrt{b(x-5)} + 10$

$-20 = \sqrt{b(-7-5)} + 10$

$-30^2 = \sqrt{b(-12)}^2$

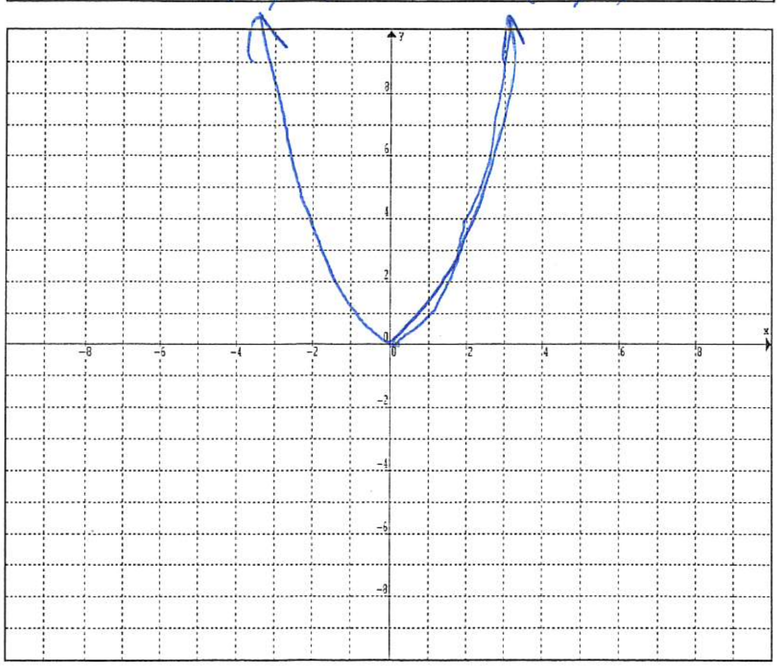
$\frac{900}{-12} = \frac{b(-12)}{-12}$

$y = \sqrt{\frac{900}{-12}(x-5)} + 10$

Graphing Quadratic Functions

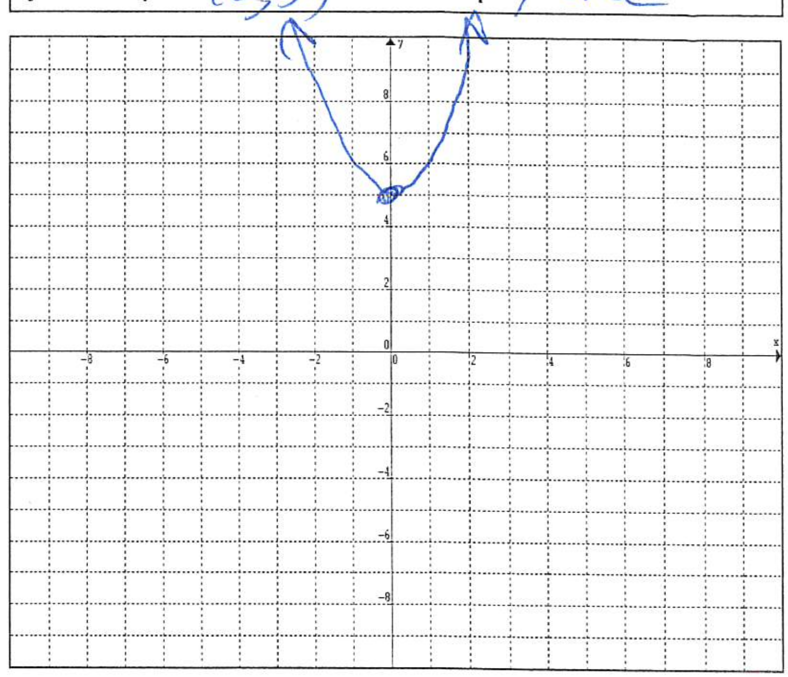
1. $f(x) = x^2$

Vertex = (0,0)
y-intercept : (0,0) x-intercept: (0,0)



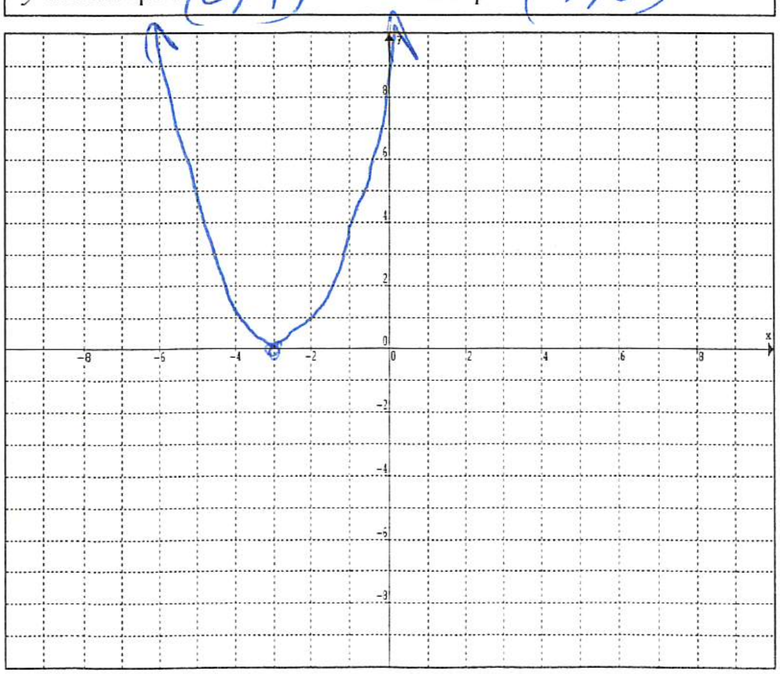
2. $f(x) = x^2 + 5$

Vertex = (0,5)
y-intercept : (0,5) x-intercept: None



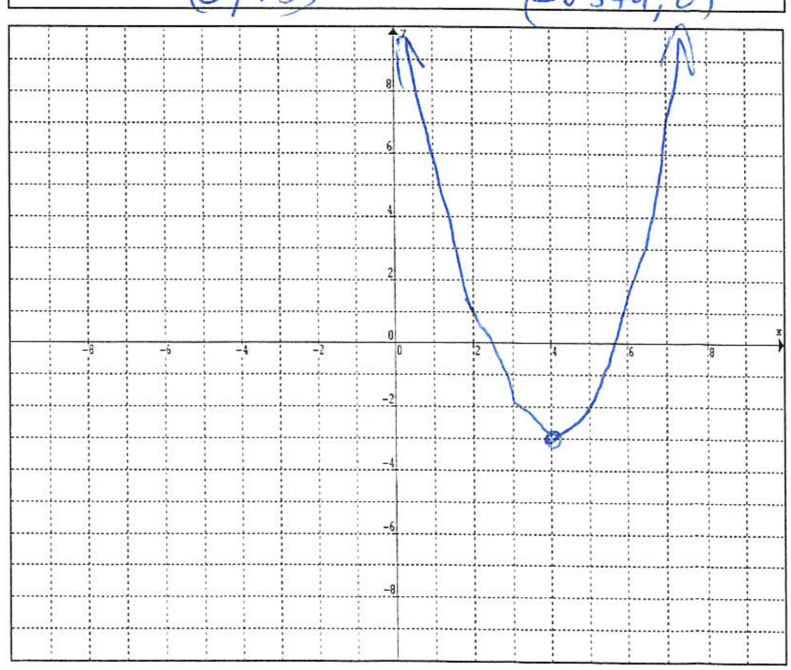
3. $f(x) = (x+3)^2$

Vertex = (−3,0)
y-intercept : (0,9) x-intercept: (−3,0)



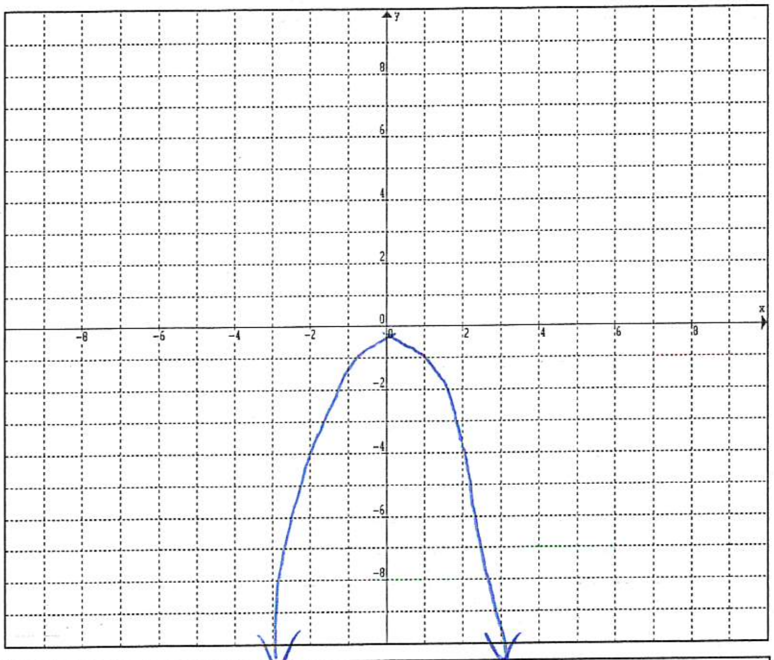
4. $f(x) = (x-4)^2 - 3$

Vertex = (4,−3)
y-intercept : (0,13) x-intercept: $(\sqrt{3}+4, 0)$ $(-\sqrt{3}+4, 0)$



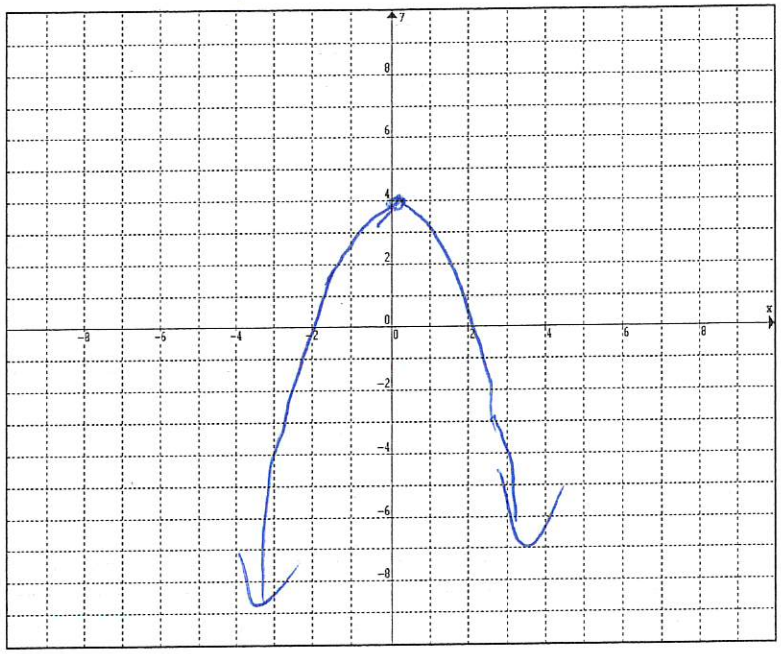
5. $f(x)=-x^2$

Vertex = (0,0)
y-intercept : (0,0) x-intercept: (0,0)



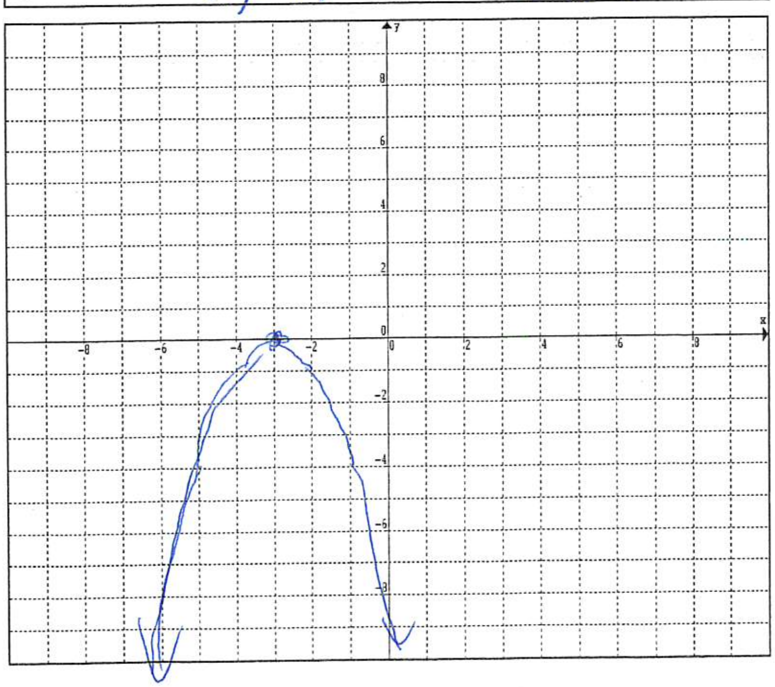
6. $f(x)=-x^2+4$

Vertex = (0,4)
y-intercept : (0,4) x-intercept: (2,0) & (-2,0)



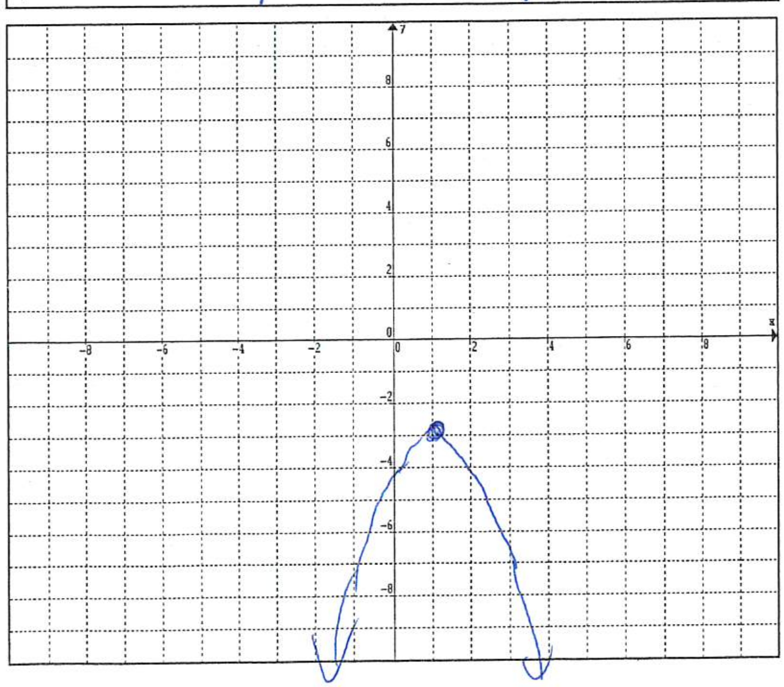
7. $f(x)=-(x+3)^2$

Vertex = (-3,0)
y-intercept : (0,-9) x-intercept: (-3,0)



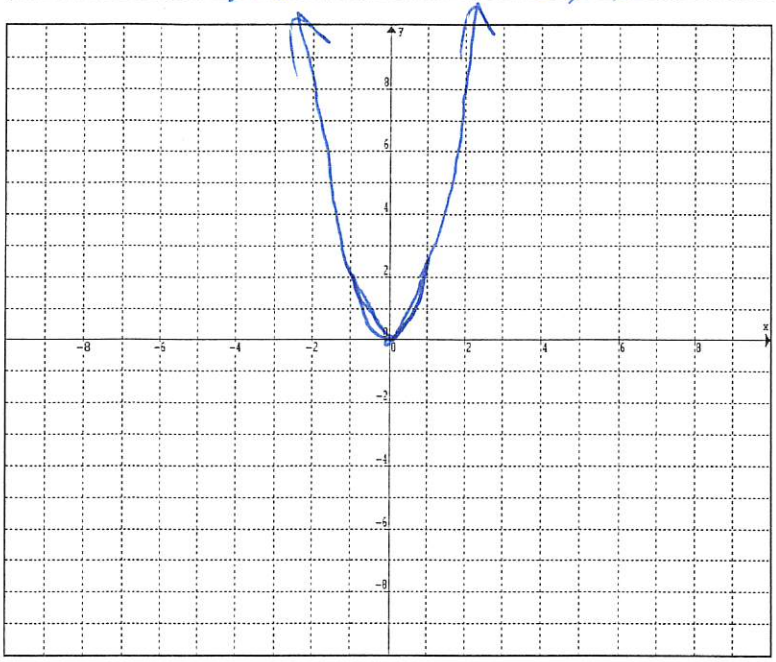
8. $f(x)=-(x-1)^2-3$

Vertex = (1,-3)
y-intercept : -4 x-intercept: None



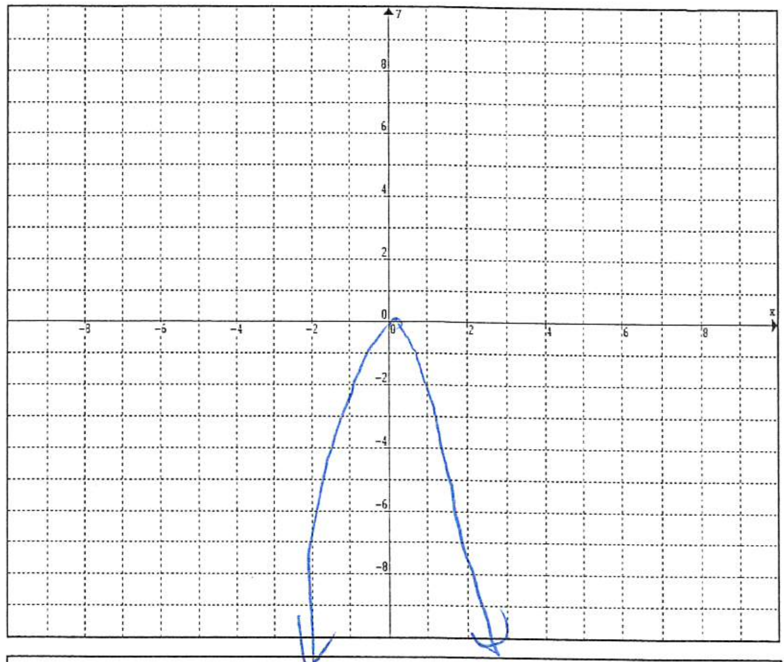
9. $f(x) = 2x^2$

Vertex = $(0,0)$
y-intercept : $(0,0)$ x-intercept: $(0,0)$



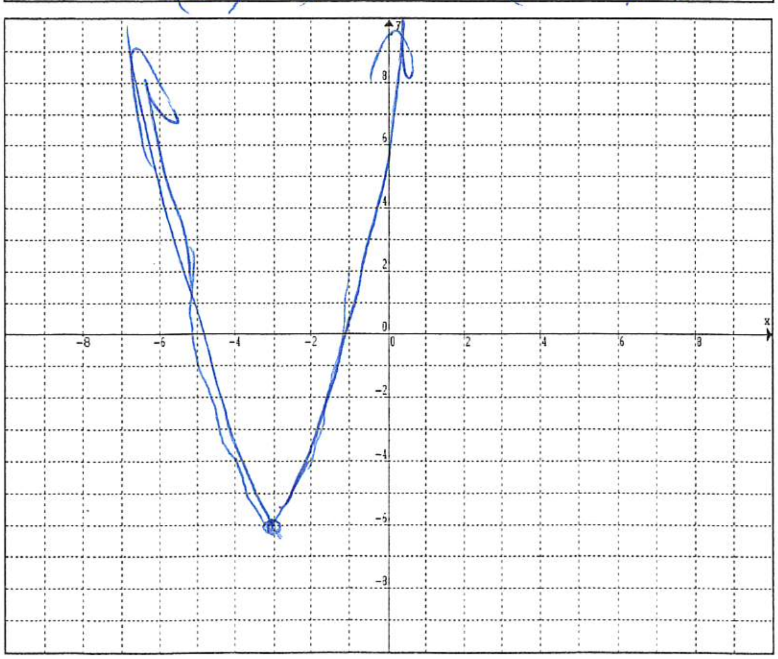
10. $f(x) = -2x^2$

Vertex = $(0,0)$
y-intercept : $(0,0)$ x-intercept: $(0,0)$



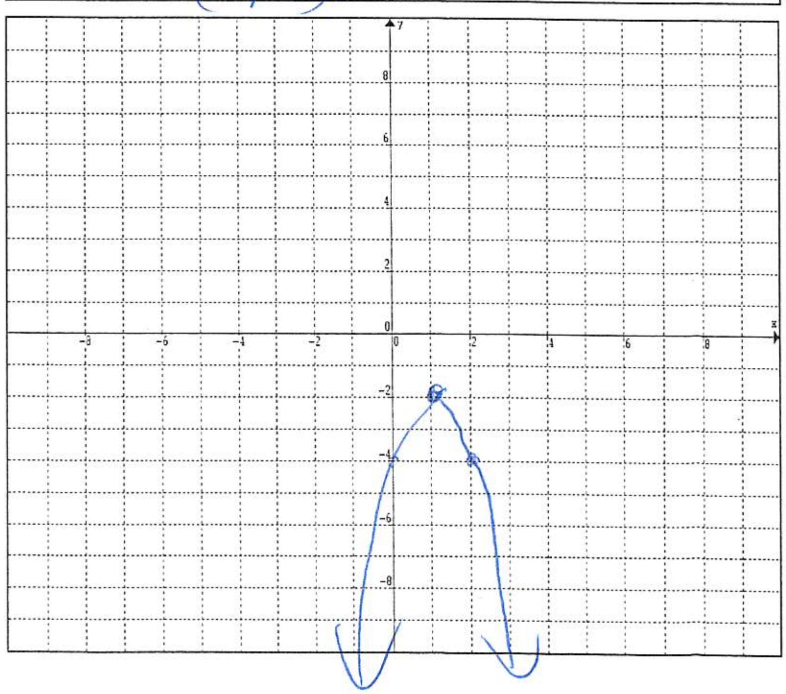
11. $f(x) = 2(x+3)^2 - 6$

Vertex = $(-3,-6)$
y-intercept : $(0,12)$ x-intercept: $(\sqrt{3}-3, 0)$ $(-\sqrt{3}-3, 0)$



12. $f(x) = -2(x-1)^2 - 2$

Vertex = $(1,-2)$
y-intercept : $(0,-4)$ x-intercept: None



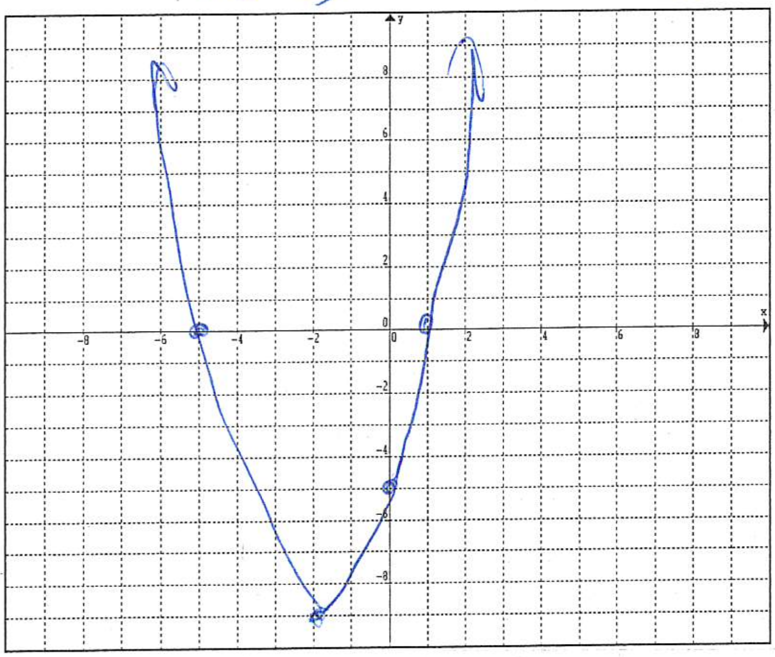
13. $f(x) = x^2 + 4x - 5$

$(x+5)(x-1)$

$(x+2)^2 - 9$

Vertex = $(-2, -9)$

y-intercept : $(0, -5)$ x-intercept: $(-5, 0)$ $(1, 0)$

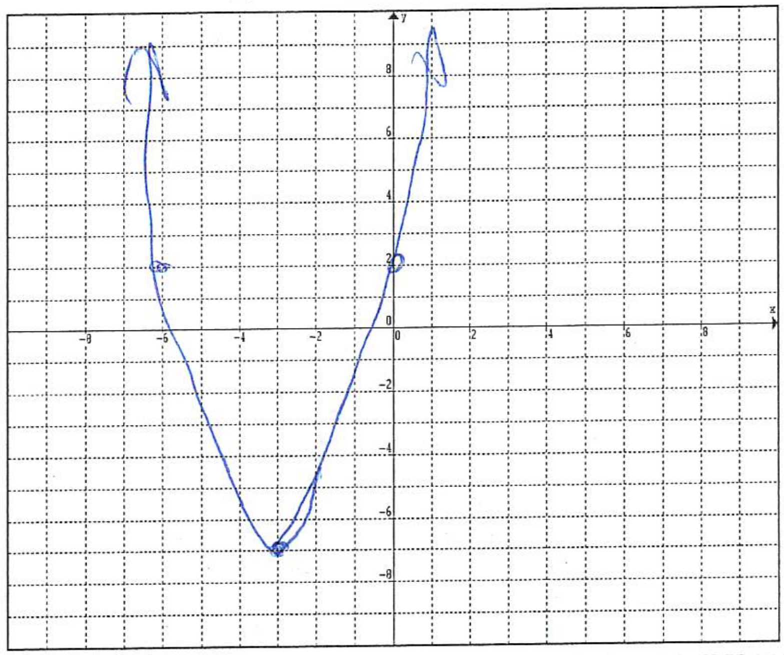


14. $f(x) = x^2 + 6x + 2$

$(x+3)^2 - 7$

Vertex = $(-3, -7)$

y-intercept : $(0, 2)$ x-intercept: $(\sqrt{7}-3, 0)$ $(-\sqrt{7}-3, 0)$

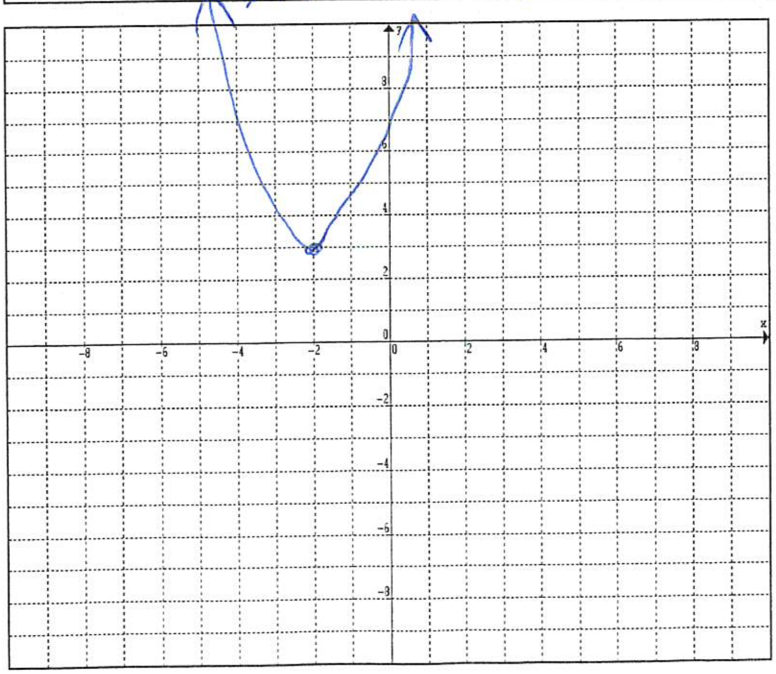


15. $f(x) = x^2 + 4x + 7$

$(x+2)^2 + 3$

Vertex = $(-2, 3)$

y-intercept : $(0, 7)$ x-intercept: None

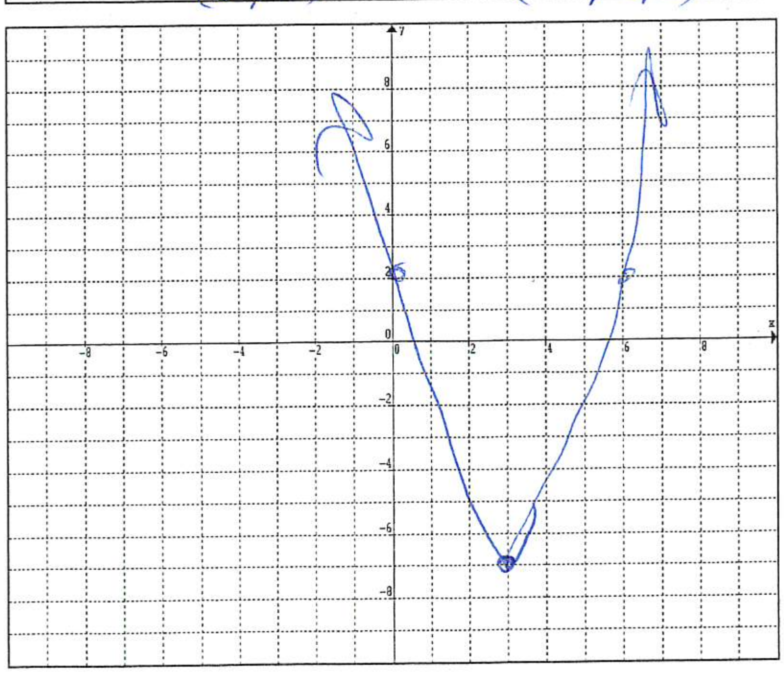


16. $f(x) = x^2 - 6x + 2$

$(x-3)^2 - 7$

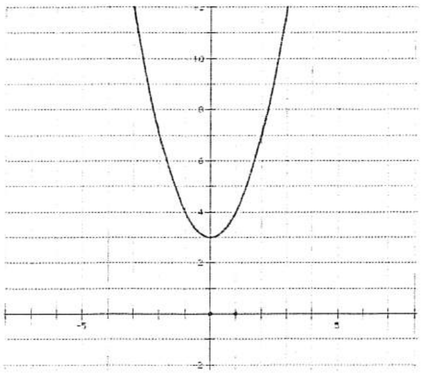
Vertex = $(3, -7)$

y-intercept : $(0, 2)$ x-intercept: $(\sqrt{7}+3, 0)$ $(-\sqrt{7}+3, 0)$

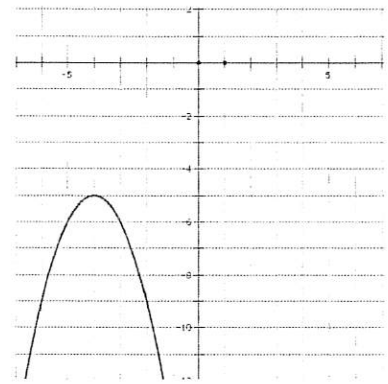


Write an equation of each graph below in the form $f(x) = a(x-h)^2 + k$.

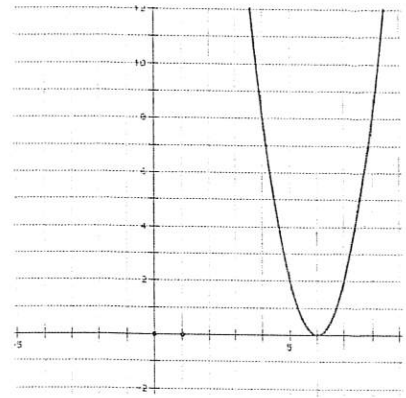
33. $f(x) =$ x^2+3



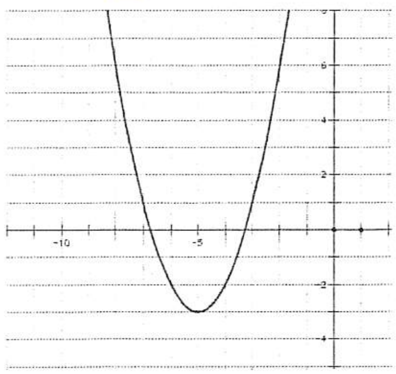
34. $f(x) =$ $-(x+4)^2-5$



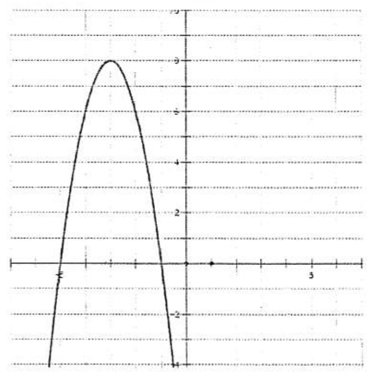
35. $f(x) =$ $2(x-6)^2$



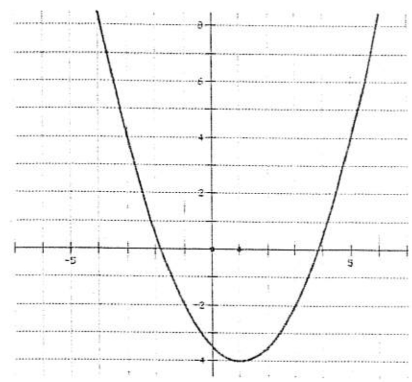
36. $f(x) =$ $(x+5)^2-3$



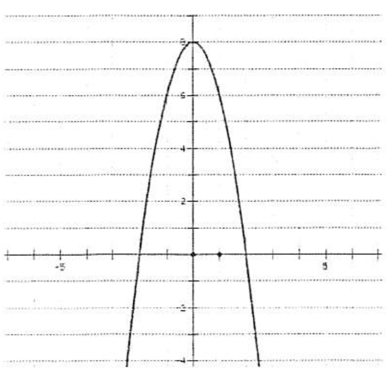
37. $f(x) =$ $-2(x+3)^2+9$



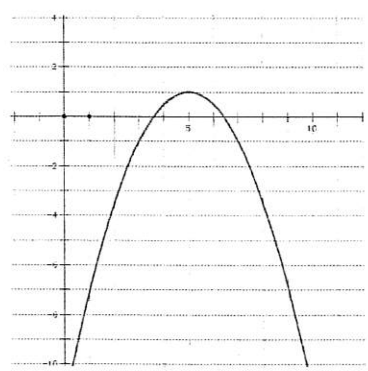
38. $f(x) =$ $\frac{1}{2}(x-1)^2-4$



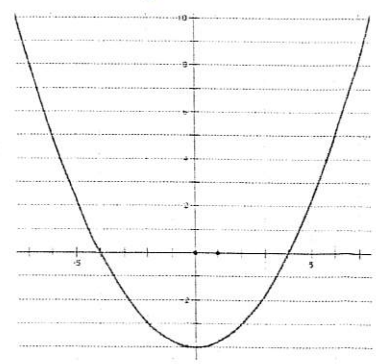
39. $f(x) =$ $-2x^2+8$



40. $f(x) =$ $-\frac{1}{2}(x-5)^2+1$



41. $f(x) =$ $\frac{1}{4}x^2-4$



1. Identify each transformation (or transformations) below. Be specific.

Transformations:		
HORIZONTAL TRANSLATION (Left or Right)	VERTICAL TRANSLATION (Up or Down)	VERTICAL REFLECTION
HORIZONTAL REFLECTION	HORIZONTAL DILATION (Stretch or Compress)	VERTICAL DILATION (Stretch or Compress)

a. $f(x)+10$
Up 10

b. $f(x-3)$
Right 3

c. $f(x+8)$
Left 8

d. $3f(x)$
Vert stretch of 3

e. $-f(x)$
vertical reflection

f. $f(0.5x)$
Horizontal dilation

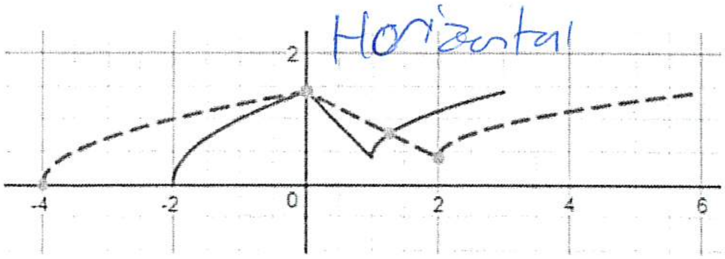
g. $f(-x)$
Horizontal reflection

h. $f(2(x-1))$
Horizontal Dilation & right 1

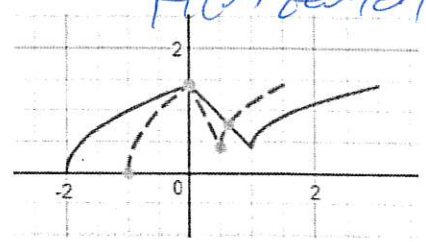
i. $f(x+3)+3$
left 3
up 3

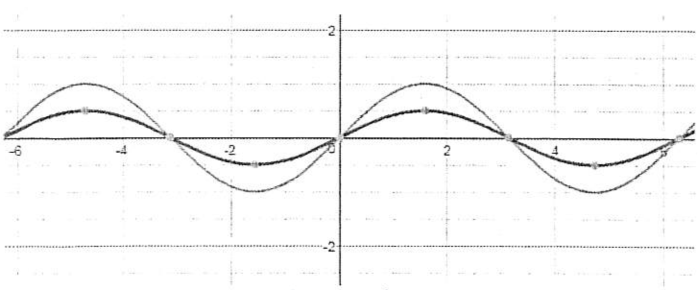
2. Which image(s) below show a horizontal dilation and which show(s) a vertical dilation? How can you tell?

Horizontal

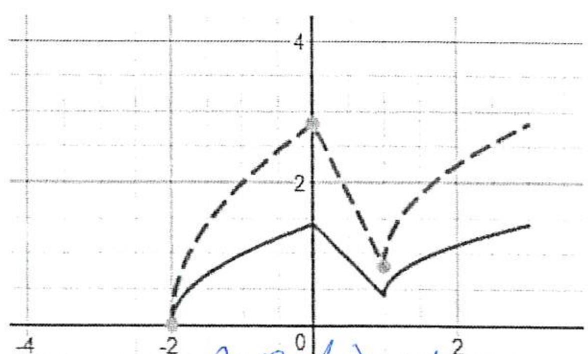


Horizontal





Vertical

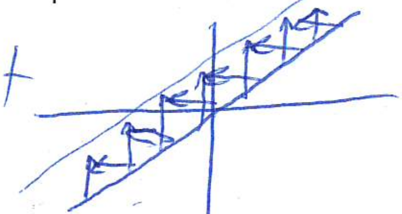


vertical

3. Let the Parent LINEAR Function be $g(x) = x$.

a. Explain GRAPHICALLY why a vertical translation up 1 unit results in the same function as a horizontal translation left 1 unit.

Because look at it



b. Will a VERTICAL REFLECTION of $g(x) = x$ look differently than a HORIZONTAL REFLECTION of $g(x) = x$? Explain how you know.

No. Same thing. $-x$ vs $(-x)$

c. Is $h(x) = 3x$ a VERTICAL or HORIZONTAL DILATION of $g(x) = x$? Explain how you know.

Vertical. No parentheses. But it also could be seen as horizontal.
3 times taller or 3 times skinnier

4. Consider the Quadratic Function $n(x) = x^2 + 10x + 21$.

a. Factor to show that $n(x) = (x + \#)(x + \#)$.

$(x+7)(x+3)$
$x=-7,\ x=-3$

b. The VERTEX is halfway between the x-intercepts. Find the x- and y-coordinates of the vertex.

$\frac{-7+-3}{2} = \frac{-10}{2} = -5$
$n(-5) = (-5+7)(-5+3)$
$= 2(-2) = -4$
$(-5, -4)$

c. What transformation(s) on $f(x) = x^2$ result in $n(x)$? Be specific.

Left 5, Down 4

d. Evaluate $n(0)$. What does $n(0)$ tell you about the GRAPH of $n(x)$?

$n(0) = 21$
Tells you the y-intercept

e. Find the VERTEX of $n(x+1) - 3$.

Left 1, Down 3
So $(-5-1, -4-3) = (-6, -7)$

1. What is the difference between the graphs of $f(x) = x^2 + 1$ and $g(x) = x^2 - 1$?

Up 1 Down 1

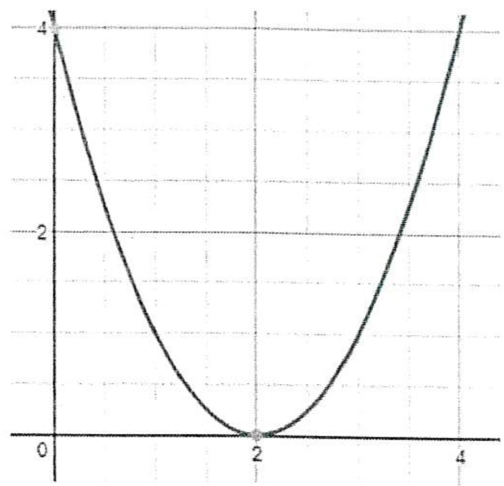
Different vertex

2. What is the difference between the graphs of $f(x) = x^2 + 1$ and $h(x) = -x^2 + 1$?

Opens up opens down

Same vertex

3. Linsey wants to create a design on desmos and started with the parabola shown. What equation did she use to create this parabola?

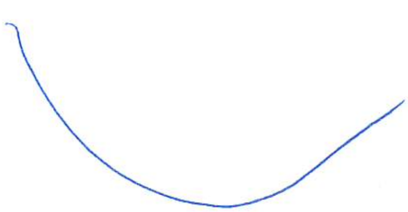


$(x-2)^2$

4. Without graphing, describe the differences in the parabolas that represent the function $p(x) = 4x^2$ and $q(x) = 0.25x^2$.

Short & wide

Tall & skinny



Vocabulary:

Parent Graph: the graphical representation of the most basic form of function family. The parent graph for Quadratic Functions is the graph of $y = x^2$.

Transformation: changes to the shape, orientation or location of the parent graph. There are three main types of transformations that we will study -- *Translation, Dilation and Reflection* (in Geometry, you also explored *Rotation*).

- Translation (Slide): moving all points on a graph horizontally or vertically a fixed amount.
  - Notation:
    - Vertical Translation $f(x)+k$
    - Horizontal Translation $f(x-k)$

- Dilation (Stretch or Compression): an increase or decrease in the height (vertical dilation) or width (horizontal dilation) of a graph by a factor. For example, $y = 3x^2$ vertically stretches the parent function by a factor of 3.
  - Notation:
    - Vertical Dilation $k\,f(x)$
    - Horizontal Dilation $f(kx)$

- Reflection (Flip): mirroring a graph over a fixed line (typically the x-axis or y-axis).
  - Notation:
    - Vertical Reflection $-f(x)$
    - Horizontal Reflection $f(-x)$

7. For each function below, describe how the parent function $f(x) = x^2$ was transformed. The first one is done as an example.

a. $y = x^2 - 3$ the parent function was translated 3 units down.

b. $y = x^2 + 10$ Up 10

c. $y = (x-4)^2$ Right 4

d. $y = 0.5x^2$ Half as tall

e. $y = (2x)^2$ Half as wide

f. $y = -(x-1)^2$ (describe both transformations)
Vertical reflection & right 1

g. $y = 5(x+2)^2 - 5$ (describe all transformations)
5 times taller, left 2, down 5

CCSS Advanced Algebra 3 More Quadratic Transformations:

1. Let $f(x) = (x+7)(x-5)$. $= x^2+2x-35$

a. What are the x-intercepts of the function? y-intercept? (-7,0)(5,0) (0,-35)

b. Consider the transformation $g(x) = f(x-2)$. What are the x-intercepts of $g(x)$? (-5,0) (7,0)

c. Consider the transformation $h(x) = 3f(x) - 1$. What is the y-intercept of $h(x)$?

(0, 3·-35-1

(0, -105-1

(0, -106)

2. For the function $w(x) = x^2 + 6x + 8$, $= (x+4)(x+2)$

d. Find the x-intercepts and the y-intercept. (-4,0) (-2,0) (0,8)

e. What transformation would be applied to $w(x)$ that would result in the new function $v(x) = (x+5)(x+7)$? Move left 3

f. Explain why the transformed function $u(x) = 5w(x)$ has the same x-intercepts as $w(x)$.

Because 5·0=0.

3. Write $f(x) = (x+7)(x-5)$

a. in standard form, $f(x) = ax^2 + bx + c$. $x^2+2x-35$

b. Complete the square (see below for notes) to write $f(x)$ in Graphing Form $f(x) = a(x-h)^2 + k$ and write the vertex of the parabola.

c. What is the vertex of $g(x) = f(x-2)$?

d. What is the vertex of $h(x) = 3f(x) - 1$?

$(x+1)^2-36$

V: (-1, -36)

c) V: (1, -36)

d) V: (-1, -109)

4. Completing the square practice (see below for notes). Write each quadratic function in Graphing Form and determine the vertex (BOTH x AND y):

a. $p(x) = x^2 + 10x - 24$ $= (x+5)^2-49$

V: (-5, -49)

b. $q(x) = x^2 - 5x + 6$ $= (x-2.5)^2-.25$

V: (2.5, -.25)

c. $r(x) = 2x^2 + 6x - 36$ $= 2(x^2+3x-18) = 2((x+1.5)^2-20.25)$

$= 2(x+1.5)^2-40.5$

V: (-1.5, -40.5)

d. $s(x) = 5x^2 - 20x + 25$

$5(x^2-4x+5)$

$5((x-2)^2+1)$

$5(x-2)^2+5$

V: (2, 5)

e. $t(x) = 10x^2 - 18x - 36$

$10(x^2-1.8x-3.6)$

$10((x-.9)^2-4.41)$

$10(x-.9)^2-44.1$

5. For each quadratic equation above, describe the transformations that would be required to go from the parent graph ($y = x^2$) to the new function. Be specific using the terms horizontal/vertical translation, reflection, dilation.

1	2	3	4.
left 1, down 36	left 3, Down 1	left 1 Down 36	Depends

Completing the Square Notes (Converting a Quadratic Function from Standard to Graphing Form):

Example: $f(x) = x^2 + 20x + 36$

Step 1: Create a generic rectangle and put the x^2 in the lower left corner.

$f(x) = \underline{x^2} + \underline{20x} + 36$

x^2	

Step 2: Split the $20x$ in half and place each half in the generic rectangle.

**Why does it make sense to do this?

$10x$	
x^2	$10x$

Step 3: Fill out the outside (base and height) of the generic rectangle.

10	$10x$	
x	x^2	$10x$
	x	10

Step 4: Complete the inside of the generic rectangle using the outside values.

10	$10x$	100
x	x^2	$10x$
	x	10

Step 5: Determine what value must be added to the generic rectangle to match the original function.

$f(x) = \underline{x^2} + \underline{20x} + 36$

10	$10x$	100	$+ \boxed{k}$
x	x^2	$10x$	
	x	10	

$100 + k = 36$

$-100 \quad -100$

$k = -64$

Step 5: Write the function in Graphing Form.

$f(x) = (x + 10)^2 - 64$