Algebra 2 Name:___________

Why is Solving a Quadratic Equation[1] So Challenging?

1. **Challenge: Think of 2 numbers that add up to equal zero. Write them down, then compare with your team:**

   12 & -12

   a. **Ok, I hope that wasn't too much of a challenge. Do you think that you found the only 2 numbers that would work? If so, explain why. If not, list a few more pairs of numbers that sum to zero.**

      Many possibilities

   b. **I hope you found that there are many pairs of numbers that sum to zero (like $3 \text{ and } -3$, $-17 \text{ and } 17$, and $-\pi$ and π). Try to generalize the pattern you have found. In other words, if "x" is one of your numbers, and the sum is zero, what must be the other number? Why?**

      x & -x

2. **Here's another challenge: Think of 2 numbers that multiply to equal zero. Write them down, then compare with your team:**

   7 & 0

   a. **Perhaps that wasn't too challenging either. Do you think that you found the only 2 numbers that would work? If so, explain why. If not, list a few more pairs of numbers with a product of zero.**

      Many numbers

   b. **I hope you found that there are many pairs of numbers with a product of zero (like $3 \text{ and } 0$, $-17 \text{ and } 0$, and 0 and π). Try to generalize the pattern you have found. In other words, if "x" is one of your numbers, and the product is zero, what must be the other number? Why?**

      x & 0

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[1] (in standard form). Quadratic equations in factored form are pretty easy to solve, and quadratic equations in vertex form aren't that bad. It's standard form that's a real doozy.

The Zero Product Property
If $x \cdot y = 0$, then either $x = 0$ or $y = 0$ (or both).

3. **Here's a harder challenge: Think of 3 numbers that sum to zero. Write them down, then compare with your team.**

   2, 3, -5

   a. **Do you think that you found the only 3 numbers that would work? If so, explain why. If not, list a few more triplets of numbers that sum to zero.**

   Many Numbers

   b. **I hope you found that there are many triplets of numbers that sum to zero (like $3\ and\ 2\ and\ -5$, $-17\ and\ 0\ and\ 17$, and $2\pi\ and\ -5\pi\ and\ 3\pi$). Try to generalize the pattern you have found. In other words, if "x" is one of your numbers, "y" is the second number, and the sum is zero, what must be the other number? Why?**

   x, y, & $-(x+y)$

4. **A quadratic[2] challenge: Think of 3 numbers that sum to zero AND where the first number is the square of the second number. The third number can be whatever you want.**

   1, 1, -2

   a. **Do you think that you found the only 3 numbers that would work? If so, explain why. If not, list a few more quadratic triplets of numbers that sum to zero.**

   Many #s

   b. **I hope you found that there are many quadratic triplets of numbers that sum to zero (like $3\ and\ 9\ and\ -12$, $-17\ and\ 289\ and\ 272$, and $.5\ and\ .25\ and\ -.75$). Try to generalize the pattern you have found. In other words, if "x" is one of your numbers, "x^2" is the second number, and the sum is zero, what must be the other number? Why?**

   x^2, x, & $-(x^2+x)$

[2] This is called "quadratic" because it involves "squaring" a number. Quadratic and square both come from the same root word meaning "four-sided."

5. A Harder Quadratic Challenge: Think of 3 numbers that sum to zero. The second number must be the square of the first number. The third number must be -6.

$x=2$ & $x=-3$

   a. It is OK if you were not able to complete that challenge. In fact, the point of this assignment is to understand WHY it is so hard to complete quadratic challenges. Explain why Challenge 5 is harder than Challenges 1-4.

   Because there aren't many numbers that work

   b. Write Challenge 5 as an equation. How many solutions do you expect your solution to have?

   $x^2+x-6=0$

   c. Many of you may have found that $x=2$ is a solution to Challenge 5, because $2^2+2-6=0$. In other words, $x=2$ satisfies the equation $x^2+x-6=0$. Explain why $x=-3$ is <u>also</u> a solution.

   B/C $(-3)^2+(-3)-6=0$

   $9-3-6=0$

6. A Second Quadratic Challenge: Think of 3 numbers that sum to zero. The second number must be the square of the first number. The third number must be -12

$x=3$ & $x=-4$

   a. Write Challenge 6 as an equation. How many solutions do you expect your solution to have?

   $x^2+x-12=0$

   b. Many of you may have found that $x=3$ is a solution to Challenge 6, because $3^2+3-12=0$. In other words, $x=3$ satisfies the equation $x^2+x-12=0$. Explain why $x=-4$ is also a solution.

   B/C $(-4)^2+-4-12=0$

   $16-4-12=0$

Quadratic Equations
Equations of the form $ax^2 + bx + c = 0$ are called STANDARD FORM.

7. **Quadratic Equations in STANDARD FORM: These are numerical challenges like the ones you just solved. Use your knowledge of problems that sum to equal zero (and the zero product property) to solve the following quadratic equations. All of the solutions are integers between -10 and 10, so there is no excuse for giving up. If you get stuck, just guess and check!**

   a. $x^2 + x - 30 = 0$

   x=5 & -6

   b. $x^2 - x - 2 = 0$

   x=2 & x=-1

   c. $x^2 + 2x - 3 = 0$

   x=-3 & x=1

   d. $2x^2 + 4x - 6 = 0$

   x=-3 & x=1

   e. $x^2 + 3x + 2 = 0$

   x=-2 & x=-1

8. **You may not have been able to solve those easily... THAT IS TOTALLY OK! The point of this activity was to learn about WHY quadratic equations in STANDARD FORM are difficult to solve. Write yourself a note about what you learned about solving quadratic equations.**

   Because x is in 2 different locations so it's hard to isolate.